# Regulation and Service Provision in Dynamic Oligopoly: Evidence from Mobile Telecommunications

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#### Abstract

I study coverage requirements, a common regulation in the mobile telecommunications industry that intends to accelerate the roll-out of new mobile telecommunications technologies to disadvantaged areas. I argue that the regulation may engender entry deterrence effects that limit its efficacy and lead to technology introduction patterns that are not cost-efficient. To quantify the impact of coverage requirements on market structure and the speed and cost of technology roll-out, I develop and estimate a dynamic game of entry and technology upgrade under regulation. I estimate the model using panel data on mobile technology availability at the municipality level in Brazil. In counterfactual simulations, I find that coverage requirements accelerate the introduction of 3G technology by just over 1 year, on average, and reduce firms' profits by 24% relative to a scenario with no regulation. I find the entry deterrence effects to be small. Moreover, an alternative subsidization policy leads to a similar acceleration in the roll-out of 3G and substantially higher aggregate profits, likely increasing aggregate welfare relative to coverage requirements.

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### 1 Introduction

In industries with large fixed costs, firms' failure to appropriate the consumer surplus they generate when they enter new markets and introduce new products may lead to underprovision of goods and services. This possibility is particularly relevant in disadvantaged areas, where the prospects of recouping fixed costs are dim. Concerns regarding service underprovision have led to regulatory oversight and intervention in many industries, such as postal service, healthcare, airlines, and telecommunications.<sup>1</sup> These concerns have historically been particularly salient in the telecommunications industry (Wu (2010)). The substantial investment costs required for network expansion raise fears that firms will not provide service and bring new mobile telecommunications technologies to low-income, rural, or isolated localities, despite the considerable benefits associated with these services.<sup>2</sup> These concerns have led to the regulation of the roll-out of new mobile telecommunications technologies in countries ranging from Nigeria to the United States. This paper studies the effects of existing regulation on the introduction of new mobile telecommunications technologies, and evaluates the desirability of existing regulation relative to alternative forms of intervention.

Mobile telecommunications markets are typically characterized by a small number of firms. To provide mobile telecommunications services, these firms must acquire from the government licenses to use the radio spectrum. These licenses tyically cover large geographic areas containing many local markets. In the absence of regulation, firms would choose to provide service and introduce new technologies in those markets where variable profits exceed fixed costs, potentially leaving some areas without service or access to new technologies. To avoid this outcome, regulators impose what are called coverage requirements. A coverage requirement tasks a single firm with providing service of a specific technology in a given area by a date set by the regulator.<sup>3</sup>

The goal of this paper is to understand the welfare effects of coverage require-

<sup>&</sup>lt;sup>1</sup>USPS is subject to a Universal Service Obligation. The HRSA runs the Medicare Rural Hospital Flexibility Program. The DOT runs the Essential Air Service and Small Community Air Service Development Program. The Universal Service Administrative Company spends almost ten billion dollars annually in subsidies for high-speed broadband access.

<sup>&</sup>lt;sup>2</sup>Telecommunications services have been shown to have positive effects on economic growth (Roller and Waverman (2001), Czernich, Falck, Kretschmer, and Woessmann (2011)); labor productivity (Bertschek and Niebel (2016), Akerman, Gaarder, and Mogstad (2015)); market efficiency (Jensen (2007)), and risk-sharing (Jack and Suri (2014)). Aker and Mbiti (2010) discuss many other potential benefits of mobile telecommunications in developing countries.

<sup>&</sup>lt;sup>3</sup>Another common form of coverage requirements is that firms are obliged to provide service to at least some fraction of the territory covered by their license by a date set by the regulator. This fraction varies across countries and in some cases is close to 1.

ments and alternative regulatory interventions. At first glance, the trade-off faced by regulators when deciding whether or not to impose a coverage requirement is clear. On the one hand, the requirement presumably accelerates the introduction of the new technology in the regulated area, thus increasing the discounted stream of consumer surplus. On the other hand, coverage requirements impose a cost on the regulated firm, for it is required to enter a market or upgrade its technology when it might not have done so in the absence of regulation. The oligopolistic structure of the mobile telecommunications industry overturns this apparent simplicity. A coverage requirement is a credible commitment to provide service on the part of the firm subject to the regulation. This commitment may deter entry by the other firms and lead to further changes in equilibrium behavior that diminish or even reverse the acceleration of the introduction of the new technology alluded to above.

To quantify the effects of coverage requirements and alternative policies, I develop and estimate an empirical dynamic game of firm entry and technology upgrade under regulation. Firms' incentives to enter a market and upgrade their technologies are determined by the incremental variable profit derived from those choices and the associated sunk costs. Therefore, an appropriate empirical model must accurately capture the key features determining those profits and costs. An important characteristic of rapidly evolving industries such as mobile telecommunications is that demand for a new technology tends to increase over time whereas the associated adoption costs tend to decrease. Also important are local market features that shape demand and costs, as well as the local market structure. To account for these key factors, I model firms' flow profits as a time-varying function of market structure and local demographic characteristics. The model also allows the costs of introducing a new technology to vary over time and across local markets.

The other crucial determinant of firms' incentives to introduce the new technology is, of course, the regulation. In the model, as in the data, in each market exactly one firm is required to provide 3G service by a date set exogenously by the regulator. I model the regulation's enforcement by assuming that the regulated firm must pay a fine in every period after the regulation deadline in which it fails to comply with the regulation. There are two dimensions to the incentives stemming from the regulation, given its asymmetric nature. First, the single regulated firm has an added incentive to introduce the new technology, to avoid triggering punishments for non-compliance. Second, the firms that are not subject to the regulation know that the regulated firm will be in the market in the future, and with the new technology. Therefore, they know that the market will be more competitive in the future, and that knowledge negatively affects their incentives to enter and introduce the new technology. The latter mechanism may give rise to a further response by the regulated firm: knowing that the unregulated firms will not enter the market and knowing that adoption costs decrease over time, the regulated firm may have an incentive to wait for costs to fall before introducing the new technology. As this discussion makes clear, capturing these mechanisms requires an equilibrium model of entry and technology adoption.

The question of how much later (or earlier) the introduction of 3G technology would have ocurred in the absence of regulation is a question about time, and thus requires a dynamic model. The nature of the regulation, which sets a deadline for the introduction of the new technology, also makes the problem dynamic (and nonstationary). These aspects justify the dynamic nature of the model.

The time-varying nature of variable profits and technology adoption costs and the regulation deadline make the environment non-stationary, a departure from most of the literature on empirical dynamic games. I also depart from the existing empirical literature on technology adoption, which applies full-solution estimation routines based on backward induction solution algorithms. I instead assume that structural parameters stabilize before the end of the sample and focus on what I call quasi-stationary Markov Perfect Equilibria (QMPE). Essentially, QMPE have a non-stationary phase followed by a stationary phase. This structure allows me to adapt existing estimation methods used in stationary dynamic games to a non-stationary setting.

I estimate the model using new panel data on mobile technology availability at the municipality level in Brazil from June 2013 to June 2020. I analyze firms' entry and technology upgrade behavior in a set of mostly rural municipalities. In each of these municipalities, exactly one of the four major carriers in the country was required to provide 3G service by a date set by the regulator. I call that firm the regulated firm. The identity of the regulated firm varied across municipalities; all of the four major carriers in the country are regulated in some markets but not others. Comparing the behavior of regulated and unregulated firms shows that the latter are less likely to enter a market or upgrade their technology when the regulated firm is yet to satisfy its coverage requirement. This pattern is consistent with the entry deterrence effect outlined above.

The model estimates show that the profits and costs associated with 3G are stable over my sample period. The profits associated with 4G rise sharply, and the costs of 4G installation decrease substantially. The latter inference is driven by a sharp increase in 4G introductions in the final part of the sample. The cost of non-compliance with the regulation is not directly observed, but it is identified from differences in behavior between regulated and unregulated firms. I estimate it to be sizeable: it amounts to about 40% of the median entry cost. Counterfactual exercises show that in the absence of coverage requirements, 3G technology would have been introduced 1.15 year later, on average. Coverage requirements accelerate the introduction of 3G in almost all municipalities, but there is substantial heterogeneity in the magnitude of that effect. For four markets, equilibrium effects imply that the regulation delays the introduction of 3G, though those effects are quantitatively small. The regulation reduces firms' aggregate expected profits by 1.2 billion 2010 USD, or 24.14% of the profits they obtain in the absence of regulation. I find the entry deterrence effects to be small; the overall effect of the regulation is almost equal to its direct effect on the regulated firm.

I also use the model to evaluate alternative policy interventions. I find that a policy that subsidizes the first firm to introduce 3G technology leads to a slightly larger acceleration of its roll-out. Moreover, firms benefit substantially from the subsidy: their aggregate profits increase by 659 million dollars, or 17% of their earnings under coverage requirements, after accounting for the financing of the subsidy. These gains stem primarily from a more cost-efficient pattern of technology adoption. The subsidy typically leads an incumbent to introduce the new technology, whereas coverage requirements are imposed on potential entrants in many cases. Incumbents only incur technology installation costs, whereas potential entrants also incur entry costs, which I estimate to be sizeable. This difference drives the cost-efficiency gains. Moreover, subsidy recipients also directly benefit from it. The cost efficiencies associated with the subsidy come at the expense of reduced competition in the market. However, I estimate that one more firm in the market has to generate a gain in consumer surplus that exceeds 40% of consumers' average expenditures for coverage requirements to be preferred to the subsidy. These results suggest that subsidization is a more efficient policy than the current form of regulation.

This paper relates to the literature studying how regulation affects market structure and market outcomes in dynamic environments. Ryan (2012) shows that stricter environmental regulation increases entry costs, thus decreasing both the number of firms in the market and consumer surplus. Gowrisankaran, Lucarelli, Schmidt-Dengler, and Town (2011) study the effect of the Medicare Rural Hospital Flexibility Program on health care provision in rural America, and show that the program expanded coverage but had a net adverse effect on consumer welfare due to provisions that limited the size and scope of regulated hospitals. Dunne, Klimek, Roberts, and Xu (2013) study the effects of entry subsidies under the Health Professional Shortage Areas program on local market structure. I contribute to this literature by studying the effect of regulation on the set of products (mobile telecommunications technologies) offered by firms and by studying the effects of asymmetric regulation. This paper also relates to the empirical literature on technology adoption. Schmidt-Dengler (2006) studies US hospitals' decisions to adopt magnetic resonance imaging (MRI). Igami (2017) studies how cannibalization, preemption, and incumbents' cost advantages shape firms' adoption of a new generation of hard disk drives. My paper adds to this literature by studying how regulation affects technology adoption. Methodologically, my work departs from the previous literature on technology adoption. Models of technology adoption must allow for time-varying demand and adoption costs. The aforementioned papers accommodate this source of non-stationarity and apply full solution estimation methods, based on backward induction algorithms. Backward induction can be applied in these settings due to a finite horizon assumption (Igami (2017)) or full adoption in finite time (Schmidt-Dengler (2006)). I instead model technology adoption as happening in an infinite horizon and assume that the game has a non-stationary part followed by a stationary part. The aforementioned notion of quasi-stationary Markov Perfect Equilibria allows me to adapt existing iterative estimation methods to this non-stationary setting.

My work also relates to the literature on regulation in telecommunications markets. Most recently, Björkegren (2019) has studied the adoption of mobile phones in Rwanda, and in that context evaluated the welfare effect of rural coverage requirements imposed on the dominant mobile network operator. His model is one of consumer choice, not firm rollout. I add to this work by modeling how firms respond to the coverage requirements, and moreover by doing so in an oligopoly context. My work also relates to an earlier, mostly theoretical, literature on universal service obligations, such as Armstrong (2001), Choné, Flochel, and Perrot (2002), and Valletti, Hoernig, and Barros (2002), that was motivated by liberalization in the telecommunications industry (and also in the postal services industry) in the 1990s. My work is the first to empirically quantify the effect of such regulation on service provision and the introduction of new technologies.

Methodologically, this paper is related to a long literature on applied dynamic games, going back to Ericson and Pakes (1995). The model I will present below will be a dynamic game with discrete controls. A number of estimators have been proposed for stationary dynamic games with discrete controls, e.g., Aguirregabiria and Mira (2007), Pakes, Ostrovsky, and Berry (2007), and Pesendorfer and Schmidt-Dengler (2008). I will depart slightly from that literature in that my model will feature a non-stationary phase followed by a stationary phase. I show that with a cross-section of markets and the notion of Quasi-Stationary Markov Perfect Equilibria, these estimators can be applied to non-stationary settings.

The rest of the paper is organized as follows. Section 2 introduces the institutional

setting, the data, and presents some preliminary evidence on the effects of coverage requirements on firm behavior. Section 3 introduces a model of entry and technology upgrade with regulated and unregulated firms. Section 4 discusses the identification and estimation of the model, and also discusses the parameter estimates. Section 6 presents the counterfactual analysis. Finally, section 7 provides concluding remarks.

# 2 Institutional Setting and Data

Operators of mobile telecommunications networks transmit data through the radio frequency spectrum, which is a public resource and is subject to government management in most countries. Starting in the 1990s, many countries have adopted auctions as their means of allocating frequency bands to firms, including mobile telecommunications service providers. In these auctions, the government sells licenses to use bands of the radio frequency spectrum. These licenses typically come with a number of conditions, chief among them the coverage requirements that are the focus of this paper.

The Brazilian mobile telecommunications market is characterized by 6 mobile network operators (MNO), i.e., carriers that operate their own network infrastructure. There is also a handful of very small mobile virtual network operators (MVNO), which are carriers that do not own their own infrastructure, and instead rent space in one of the MNO's infrastructure. Of the 6 MNOs, four provide service in all of the country and have held licenses covering the entire Brazilian territory since the introduction of mobile telecommunications in the country. The other two MNOs provide more localized service. There has been no entry or exit in this market in the past twenty years<sup>4</sup>.

The Brazilian government conducted its first spectrum auction in 2007 and has since then imposed coverage requirements on the winners of these auctions. For the purpose of this paper, a coverage requirement is an imposition that a firm provide service in some well defined market by a deadline set by the regulator and with a minimum technological requirement (e.g., the firm may be required to provide 4G service, or either 3G service or 4G service). In Brazil, the relevant market for the implementation of the regulation is a municipality, and the requirement is considered to be satisfied if that firm provides the designated service in 80% of the municipality's territory. The details of the coverage requirements are a function of municipality pop-

<sup>&</sup>lt;sup>4</sup>In the last couple of years, a process of consolidation has started. Nextel, one of the two small MNOs was sold to Claro, one of the large ones. Oi, one of the big firms, is in the process of being sold, most likely to a consortium formed by the other three large MNOs.

ulation. In municipalities with more than 100,000 inhabitants, 4 MNOs were required to provide 3G service by April 2013; in municipalities with population between 30,000 and 100,000, 3 MNOs were required to provide 3G service by the end of 2017; and in municipalities with population below 30,000, 1 MNO was required to provide 3G service.<sup>5</sup> For the latter group of municipalities, there were four different deadlines: April 2014, April 2016, June 2017, and December 2019.

I focus on the group of municipalities with less than 30,000 inhabitants. The coverage requirements targeting these municipalities are the most likely to influence the availability of service, for in larger municipalities it is probable that firms would have sufficient incentives to enter the market by themselves<sup>6</sup> I will speak of the single firm in each of these markets that is subject to a coverage requirement as the *regulated firm*; I will refer to the other firms as the *unregulated firms*. All the MNOs are regulated in some markets, but not all. Though these coverage requirements target the introduction of 3G technology, the regulated firm is considered to comply with the regulation if it deploys 4G technology instead. The descriptive analysis in this section uses data from all the municipalities with less than 30,000 inhabitants<sup>7</sup>. The structural analysis will focus on the subset of municipalities with a December 2019 deadline<sup>8</sup>.

The motivation for coverage requirements rests on two premisses. First, mobile telecommunications services generate substantial welfare gains<sup>9</sup>. In the words of the Brazilian telecom regulator<sup>10</sup>:

[Mobile telecommunications technologies] *create employment opportunities, improve the education system, increase firm productivity, allow access to public digital services, among other benefits.* 

Second, for the intervention to be justified, it must be that firms do not internalize the entirety of the surplus generated by their entry and introduction of new technologies. This seems likely, given the multiple aspects of these benefits and firms' limited ability

<sup>&</sup>lt;sup>5</sup>There are also coverage requirements related to 4G technology, but those only apply to municipalities with population above 30,000. There is no 4G coverage requirement in the municipalities with less than 30,000 inhabitants, which are the ones I focus on.

<sup>&</sup>lt;sup>6</sup>It is likely that the coverage requirements targeting larger municipalities affect the number of firms in the market, but not the availability of service, which is the focus in this paper.

<sup>&</sup>lt;sup>7</sup>The sample used here is subject to a single sample selection criterion. The regulator has provided me with two different sources of information on the identify of the regulated firm in each market. I keep only the municipalities where these two sources agree with each other.

<sup>&</sup>lt;sup>8</sup>This is mostly for computational convenience, as in the structural model the definition of the state space depends on the regulation deadline. A revision of this paper will incorporate data from the other municipalities with less than 30,000 inhabitants.

<sup>&</sup>lt;sup>9</sup>See, e.g., the references in footnote 2

<sup>&</sup>lt;sup>10</sup>See, https://www.anatel.gov.br/setorregulado/telefonia-movel (last accessed in October 22, 2020).

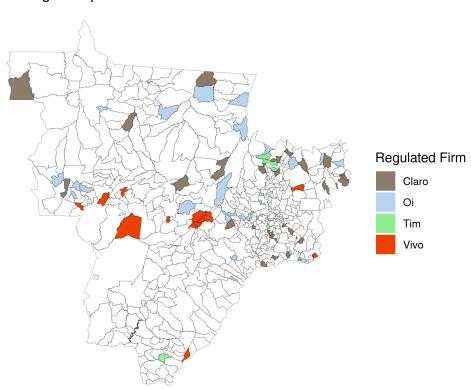
to price discriminate.

Coverage requirements are enforced by the regulator in a number of ways. First, carriers are required to deposit financial guarantees with the regulator; these guarantees can be executed if the carrier fails to satisfy its coverage requirements. Perhaps more importantly, if a carrier fails to satisfy its coverage requirements, its license can be revoked. In this case, the carrier would also be charged the value paid for its license in proportion to the time used.

The selection of which carrier was to hold the coverage requirement in each municipality was subject to a number of rules. First, the country was divided into 131 "service areas". These varied substantially in size, from a single municipality to an entire code area, which include on average 83 municipalities. Within each of these service areas, one of the four large carriers was required to select 2.5% or 5% of the municipalities in that service area that were subject to the 3G coverage requirements imposed in 2012. The fraction of municipalities to be chosen depended on the license acquired by the firm. The carriers would take turns until all municipalities were chosen. Whenever the number of remaining municipalities in a service area was too small for this rule to be feasible, the regulator decided how many municipalities each carrier would have to choose. Figure 1 shows the result of this process. The figure shows a map of the Brazilian midwest, color-coded according to the identity of the regulated carrier. Each subdivision in the map is a municipality. The municipalities with no color were not the subject of the 2012 coverage requirements. All the municipalities in color had to be chosen by some carrier. The noteworthy feature of this figure is that there is no obvious clustering; the municipalities where a firm is regulated are fairly spread out over the map.

The main dataset used in this study comes from ANATEL, the Brazilian telecommunications regulator. The data records at a monthly frequency, for each of the 5,770 municipalities, and for each of the country's mobile network operators whether or not they provide 2G, 3G, and 4G service in that municipality.<sup>11</sup> Figure 2 illustrates the structure of the data. The figure shows mobile technology availability in the state of Pará, a relatively poor northern state of Brazil. Each column of the figure corresponds to one of the four major carriers in Brazil and each row corresponds to a year. Within each map, the smaller subdivisions are municipalities in the state of Pará. Municipalities are color coded according to the most advanced technology offered by the corresponding carrier in December of the corresponding year. Therefore, the map in the first row and first column shows the technologies offered in each municipality of

<sup>&</sup>lt;sup>11</sup>The data does not include MVNOs.



Coverage Requirements -- Midwest

Figure 1: Regulated Carriers – Midwest

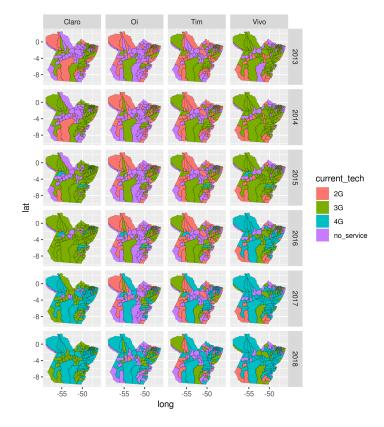


Figure 2: Technology availability in the state of Pará

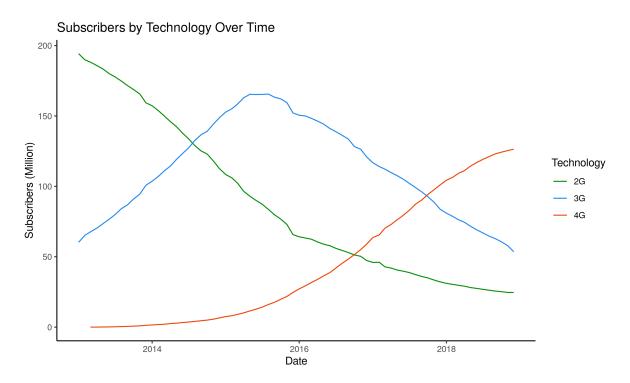


Figure 3: Subscribers by technology over time

The figure shows the total number of subscribers in the country, by technology. These quantities are calculated from ANATEL's data on subscription to mobile telecommunications services.

the state of Pará by the mobile service provider Claro in December 2013.

The second important piece of data coming from ANATEL is the identity of the regulated firm in each municipality. Finally, ANATEL also provides data on subscription to mobile telecommunications services. These data are available at the code area-month-carrier-technology level<sup>12</sup>, starting in February 2005 and until December 2018. Figure 3 shows the total number of subscribers in the country by technology for the period Jan 2013-Dec 2018. The figure shows that 2G has been in decline over the period, initially being overtaken by 3G. Moreover, 3G reaches a peak in the number of subscribers towards the end of 2015, around the time when the growth of 4G accelerates. To the extent that these patterns are driven by consumer preferences, they shape firms' incentives to introduce new technologies. The empirical model introduced below will account for this pattern in demand by allowing the demand side parameters to vary over time.

I complement the ANATEL data with a number of datasets from the Brazilian Census. First, I utilize municipality demographics and characteristics, such as population,

<sup>&</sup>lt;sup>12</sup>A code area in Brazil is much coarser than a municipality. There are 67 code areas in Brazil, and 5,770 municipalities.

GDP per capita, and area. Summary statistics on these variables are shown in table 2. Second, I use the 2017-2018 Family Budget Survey<sup>13</sup>, which provides information on households' income and their expenditure on mobile telecommunications services, among other household characteristics. Third, I use the 2010 Population Census to obtain information on the distribution of individual level demographics at the municipality level.

Table 1: Summary Statistics - Municipality Characteristics

	Variable	Ν	Mean	Std. Dev.	p10	p90
1	GDP Per Capita	972	10,936.05	10,016.44	4,248.89	20,573.72
2	Population	972	4,724.12	2,874.62	2,171.06	8,930.41
3	Area	972	1,029.50	3,799.37	90.34	1,746.49

The data in this table comes from the Brazilian Census Bureau. GDP per capita is in 2010 BRLs. Area is in squared kilometers. The values of GDP per capita are averages of data for 2010-2017, deflated to 2010 BRL. The values of population are averages of 2012-2019 data.

Table 2: Summar	y Statistics -	- Mobile Exper	nses and HH Characteristics
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	Variable	Ν	Mean	Std. Dev.	p10	p90
1	Mobile Spending	77,655	88.23	166.48	6.74	259.82
2	HH Income PC	77,655	1,687.10	1,556.25	507.56	3,348.01
3	No. Residents	77,655	2.21	1.04	1	4
4	Urban	77,655	0.81	0.39	0	1

The data in this table comes from the 2017-2018 Family Budget Survey. The unit of observation is an individual. Mobile spending is the total amount the individual spent on mobile telecommunications. It is the sum of expenditures on voice and data plans, pre-paid expenditure, and SIM cards. "HH Income PC" is the per capita income in the individual's household. "No. Residents" is the number of residents in the individual's household. "Urban" is a dummy that is equal to 1 if the individual lives in an urban area.

I drop all code areas where any of the three smaller carriers had a market share of at least 5% at any point in time. I then focus on the four major carriers. Moreover, ANATEL provides two different sources of information on coverage requirements, and I restrict attention to those municipalities for which the two sources of information are consistent with one another. The resulting sample used in the structural analysis contains 972 municipalities. Furthermore, because entering a market or upgrading a

<sup>&</sup>lt;sup>13</sup>Pesquisa de Orçamentos Familiares.

technology is a non-trivial investment that likely involves some time to build, I use data on a semester frequency rather than monthly. The unit of observation is thus a municipality-carrier-semester; there are 46,656 observations.

Table 3 shows summary statistics of the data, measured in June 2013 and December 2018, respectively. The tables show statistics for the number of active firms, the number of (firm, technology) pairs available (labeled "products" in the table), whether 3G and 4G are available, and whether the regulated and some unregulated firm offer 3G or 4G technology. In June 2013, there is on average just over 1 firm per market, and about 1.4 products; 3G is available in 28% of municipalities and 4G is not available anywhere. The regulated firm has adopted 3G technology in just over 20% of cases. In 7% of municipalities, an unregulated firm has adopted 3G.

Panel A – June 2013								
Statistic N Mean St. Dev. Min Pctl(25) Pctl(75) M								
Number of Firms	972	1.124	0.469	0	1	1	4	
Number of Products	972	1.404	0.730	0	1	2	6	
3G Available	972	0.277	0.448	0	0	1	1	
4G Available	972	0.000	0.000	0	0	0	0	
Regulated 3G+	972	0.212	0.409	0	0	0	1	
Unregulated 3G+	972	0.068	0.252	0	0	0	1	
		Panel B	– Decembe	er 2018				
Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max	
Number of Firms	972	1.665	0.655	0	1	2	4	
Number of Products	972	3.188	1.550	0	2	4	10	
3G Available	972	0.881	0.324	0	1	1	1	
4G Available	972	0.580	0.494	0	0	1	1	
Regulated 3G+	972	0.807	0.395	0	1	1	1	
Unregulated 3G+	972	0.414	0.493	0	0	1	1	

Summary statistics across the 972 municipalities in the sample, measured in June 2013 and December 2018. Number of firms is the number of firms active in a municipality. Number of products is the number of (firm,technology) pairs available in a municipality. 3G Available is a dummy that is equal to 1 if at least one firm provides 3G service in the municipality. 4G Available is defined analogously, but for 4G technology. Regulated 3G+ is a dummy that is equal to 1 if the regulated firm provides either 3G or 4G service in the municipality. Unregulated 3G+ is a dummy that is equal to 1 if some unregulated firm provides either 3G or 4G service in the municipality.

By December 2018, there are just under 1.7 firms per municipality, with about 3.2 products. By December 2018, 3G has reached 88% of municipalities, whereas 4G has

reached just under 60% of municipalities. The diffusion of new mobile technologies is driven mostly by regulated firms, but the contribution of unregulated firms is far from negligible: by December 2018, regulated firms have introduced 3G technology (or 4G) in just over 80% of the municipalities in the sample; in 41% of those municipalities, at least one unregulated firm has introduced 3G technology or better.

The descriptive statistics in table 3 suggest an important role for coverage requirements in explaining the diffusion of new mobile telecommunications technologies: regulated firms introduce 3G technology (or better) at a faster pace than unregulated firms. This difference is potentially composed of two different effects of coverage requirements: a positive effect on regulated firms and a negative effect on unregulated firms. Unregulated firms may be less likely to enter new markets or upgrade their technologies because they know that the regulated firm will introduce 3G by the requirement deadline. This implies that the market will be more competitive in the future, reducing the incentives for the unregulated firm to enter the market or upgrade its technology.

The data allow me to investigate these positive and negative mechanisms further. I estimate logit models of entry and technology upgrade decisions, which are reported in table 4. These models use data on all municipalities with a 3G coverage requirement. An observation in these models is a firm-municipality-date triple. The key explanatory variables in these models are the dummy variables "Regulated", "Regulated Competitor - Out", and "Regulated Competitor - 2G". The first of these variables is equal to 1 when the firm is regulated, and 0 otherwise. The second variable is equal to 1 when the firm faces a regulated competitor that is out of the market. The third variable is equal to 1 when the firm faces a regulated competitor that has 2G technology. The omitted case is when no firm is subject to the regulation<sup>14</sup>. The models also control for the municipality's GDP per capita, population, and area, and also include the number of competitors with each technology.<sup>15</sup> Moreover, to account for unob-

<sup>&</sup>lt;sup>14</sup>To be precise, the ommitted case pools together observations where either the regulated firm has satisfied its coverage requirement or the regulated firm is one of the small firms. Because I restrict the sample to regions where the small firms have always had negligible market shares, I interpret both situations as there being no firm subject to the regulation.

<sup>&</sup>lt;sup>15</sup>It may also be expected that a firm's network infrastructure in neighboring municipalities is important for the their choices. I test for that in Appendix B. There I do find that having service in a neighboring municipality increases the probability of entry and technology upgrade. However, the inclusion of those variables changes the estimated coefficients on the other variables only slightly, if at all. This suggests that the choice of the regulated firm is uncorrelated with their local network infrastructure. Characteristics of a firm's network in neighboring municipalities will not be included in the structural model, as doing so would increase the computational burden by several orders of magnitude. The descriptive results discussed here, however, suggest that this omission will not bias my inference regarding the effect of coverage requirements.

served municipality level heterogeneity, these models also include group fixed effects, where the groups are defined by a heuristic approach explained in detail in the Section 4.<sup>16</sup>

Each column in Table 4 corresponds to a different state for the firms included in the sample. The first column includes only observations such that the corresponding firm is not active and includes only data for the years 2013-2015; the second column includes only observations such that the firm is inactive and only data for 2016-2018; the third column includes observations such that the firm offers only 2G technology and data for 2013-2015; the samples for the remaining two columns are similarly defined. The dependent variable for columns 1 and 2 is a dummy that is equal to 1 if the firm enters the market in the next period; the dependent variable for the remaining columns is a dummy that is equal to 1 if the firm upgrades its technology in the following period. There are two key results in Table 4. First, regulated firms that have not satisfied their coverage requirements are more likely to enter the market and upgrade their technologies than unregulated firms. Second, unregulated firms are less likely to enter and upgrade their technologies when the regulated competitor is either out of the market or has 2G technology. These results show that the regulation indeed accelerates the introduction of the new technology by regulated firms, but also that it *delays* the introduction of new technologies by unregulated firms, which is evidence of the entry deterrence effects outlined in the introduction. Determining which of these two effects dominates and whether or not coverage requirements accelerate the introduction of new technologies is part of the analysis to follow.

The rest of the paper is concerned with developing tools that allows us to quantify the net effect of coverage requirements on the time to adoption of new mobile telecommunications technologies, as well as the entry deterrence effects alluded to above and the costs that the regulation imposes on firms. This requires, we will need a model of how firms make their entry and upgrade decisions. That is the topic of the next section.

## 3 Model

In this section, I introduce an empirical model of mobile service providers' decisions to enter a market and upgrade their technologies. The model operates at the level of

<sup>&</sup>lt;sup>16</sup>The group fixed effects affect the coefficients on the numbers of competitors the most. The other coefficients change only slightly with their introduction. Appendix B shows the results obtaining estimating these models without the group fixed effects.

		Dep	pendent varial	ble:	
			Upgrade		
	Out 13-15	Out 16-18	2G 13-15	2G 16-18	3G
	(1)	(2)	(3)	(4)	(5)
Log GDP PC	1.750***	0.970***	0.685***	0.195***	0.181***
	(0.091)	(0.118)	(0.066)	(0.071)	(0.038)
Log Pop.	2.495***	1.997***	1.324***	0.945***	-0.073
0	(0.104)	(0.147)	(0.072)	(0.083)	(0.045)
Log Area	-0.507***	-0.386***	-0.291***	-0.322***	0.018
C .	(0.037)	(0.050)	(0.026)	(0.030)	(0.019)
Regulated	1.735***	2.192***	2.127***	0.870***	-0.397**
0	(0.108)	(0.126)	(0.076)	(0.107)	(0.040)
Regulated Competitor - Out	-0.705***	-1.082***	0.116	-0.341**	-0.162
	(0.172)	(0.284)	(0.151)	(0.165)	(0.133)
Regulated Competitor - 2G	0.103	-0.101	-0.522***	-1.199***	-2.333**
0	(0.112)	(0.192)	(0.121)	(0.316)	(0.235)
No. Competitors 2G	-1.345***	-1.043***	-0.422***	-0.238***	$-0.064^{*}$
-	(0.093)	(0.117)	(0.055)	(0.067)	(0.038)
No. Competitors 3G	-1.937***	-2.179***	-0.598***	-0.578***	0.211***
-	(0.120)	(0.144)	(0.082)	(0.086)	(0.047)
No. Competitors 4G	-1.472	-1.534***	-2.000***	-0.889***	0.426***
-	(1.036)	(0.151)	(0.723)	(0.089)	(0.047)
Group FE	Yes	Yes	Yes	Yes	Yes
Observations	36,230	31,620	24,753	14,002	39,923

# Table 4: Entry/Upgrade Models

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

a municipality. In the model, firms' flow profits depend on their own technologies, their competitors' technologies, and the local distribution of consumers' demographic characteristics. Inactive firms make irreversible entry decisions, and both entrants and incumbents choose what technologies to offer in the market; firms incur sunk costs of entry and technology upgrade. Because one of the goals of this paper is to understand the effectiveness of coverage requirements as a tool to accelerate the diffusion of new mobile telecommunications technologies, coverage requirements are explicitly modeled. In each market a single firm is required to provide 3G technology by an exogenously specified deadline. If it fails to do so, it pays a fine every period, until it does introduce 3G technology into the market.

There are four carriers in each market. The four carriers compete by choosing which technology to operate, if any. The available technologies are 2G, 3G, and 4G. I assume that firms offer every technology less advanced than their best technology.<sup>17</sup> Time is discrete and the horizon is infinite. Within a period, the timing of the game is as follows. In the beginning of each period t incumbent firms earn their flow profits. Each firm then privately observes action-specific cost shocks, and firms simultaneously decide which of the available actions to take. Potential entrants can enter with any technology and incumbents can choose to upgrade to any technology that is more advanced than their current technology. After choosing an action, firms pay the associated costs. Technologies change deterministically according to firms' decisions.

Let  $s_{fmt}$  denote firm f's technology in market m and period t:  $s_{fmt} \in S := \{0, 2, 3, 4\}$ , where  $s_{fmt} = 0$  denotes that firm f is out of the market and the other values correspond to each of the available technologies, namely 2G, 3G, and 4G. The market's *technological state*  $s_{mt} \in S^4$  is a vector recording each firm's technology. Firms' flow profits are given by a time-varying function of the market's technological state s and the distribution  $H_x^m$  of demographics x in market m:  $\pi_t(s, H_x^m)$ . The specification of  $\pi_t$  is given in subsection 3.3.

Entry and upgrade are costly. I will allow the costs of technology introduction to vary over time in a coarse manner. I group periods into two phases; an early phase denoted E (up to and including December 2015) and a later phase denoted L (after December 2015). This allows the model to capture firms' incentives to wait for costs to decrease before introducing a new technology. I will denote by p(t) the phase associated with period t. I model the costs of deploying each technology as a technology-phase

<sup>&</sup>lt;sup>17</sup>This assumption is broadly consistent with the data. Mobile service providers typically keep old technologies in place as a fallback option. This assumption also reduces the dimension of the state space considerably, making the model computationally tractable.

specific linear function of market characteristics,  $z_m$ . Specifically, costs are modeled as

$$c_t(a, s_{fmt}, z_m, \varepsilon) = \begin{cases} -\varepsilon(a) & \text{if } a = s_{fmt} \\ \sum_{\{g': g' > s_{fmt}\}}^a z'_m \theta_{g', p(t)} + \mathbf{1} \left(s_{fmt} = 0\right) z'_m \theta_e - \varepsilon(a) & \text{if } a > s_{fmt} \end{cases}$$
(1)

In equation 1,  $a \in \{s_{fmt}, \ldots, 4\}$  is the action chosen by the firm and  $\varepsilon(a)$  is an actionspecific cost shock;  $\varepsilon$  is a vector collecting all the  $\varepsilon(a)$ . If  $a = s_{fmt}$ , the firm pays no costs (other than receiving the cost shock). A potential entrant that decides to enter pays an entry cost  $z'_m \theta_e$ . Moreover, associated with every technology g there are installation costs  $z'_m \theta_{g,p(t)}$ . One can interpret  $z'_m \theta_e$  as the cost of installing basic infrastructure, such as cell phone towers. Because it is associated with basic infrastructure, this cost does not vary over time. The term  $z'_m \theta_{q,p(t)}$  captures the cost of installing technology-specific infrastructure, such as radios that only transmit 3G or 4G signal. Because this term is associated with new technologies, it is allowed to vary over time. In equation (1),  $z_m$  is a vector of observed market characteristics and the  $\theta$ 's are parameters to be estimated. The summation in equation (1) reflects the previous assumption that firms offer all technologies less advanced than their best technology. If, for example, a firm's current best technology is 2G, and that firm upgrades to 4G, equation (1) says that the firm will pay the costs of installing both 3G and 4G.<sup>18</sup> The cost shocks are assumed to follow a Type 1 Extreme Value distribution with scale parameter  $\sigma$ , and they are iid across firms, periods, and actions.

In each market m, exactly one firm is required to provide 3G service or better by a date  $T_m$  exogenously specified by the regulator.<sup>19</sup> I will call that firm the *regulated* or *committed* firm and the other firms the *unregulated* or *uncommitted* firms. If the regulated firm fails to provide at least 3G service by the date  $T_m$ , it pays a fine  $\varphi$  every period, starting in  $T_m + 1$  and until it deploys either 3G or 4G.

Firms choose their actions to maximize their discounted expected profits, taking their competitors' behavior as given. I focus on Markov Perfect Equilibria (MPE), as is typical in empirical applications of dynamic games. I allow regulated and unregulated firms to behave differently, but beyond that I impose symmetry.

There are two sources of non-stationarity in this environment. First, flow profits and entry and technology upgrade costs vary over time. Second, coverage requirements also imply that firm behavior depends on the date. Suppose that the regulated

<sup>&</sup>lt;sup>18</sup>Note that this implies that an entering firm will always offer 2G. Because the cost of installing 2G is only paid by an entering firm,  $\theta_e$  and  $\theta_{2G}$  will not be separately identified. Therefore, in estimation I drop  $\theta_{2G}$ . The estimate of  $\theta_e$  thus includes both entry costs and 2G installation costs.

<sup>&</sup>lt;sup>19</sup>In the empirical application,  $T_m$  is always equal to December 31, 2019.

firm has not satisfied its commitment and  $t < T_m$ ; as time goes by, the regulated firm gets closer to being fined and therefore should become more likely to introduce 3G technology. Conditional choice probabilities thus change over time. I now discuss symmetry and non-stationarity in turn.

#### 3.1 Symmetric Markov Perfect Equilibria

A Markov Perfect Equilibrium is a strategy profile  $(\sigma_1, \ldots, \sigma_4)$ , such that  $\sigma_i$  is a function that maps a firm's state variables into a feasible action. In a symmetric Markov Perfect Equilibrium, strategies don't depend on firms' identities. Instead, I define value and policy functions for regulated and unregulated firms. To simplify the notation, I subsume all the market-specific variables that do not vary over time in a superscript. The state of an unregulated firm is  $(s_1, s_r, s_-, t, \varepsilon)$ , where  $s_1$  is that firm's technology,  $s_r$  is the technology of the regulated firm, and  $s_-$  is a vector with the technologies of the other two firms. The state of a regulated firm is  $(s_1, s_-, t, \varepsilon)$  where now  $s_-$  denotes the technologies of the three remaining firms. Let  $\Omega_0, \Omega_1$  denote the state space for unregulated and regulated firms, respectively, with typical element  $\omega_r, r \in \{0, 1\}$ . A strategy is a function  $\sigma_r : \Omega_r \to \{0, 2, 3, 4\}$  satisfying the restriction that  $\sigma_r(\omega_r) \in A(s_1(\omega_r)) := \{s_1(\omega_r), \ldots, 4\}$ , where  $s_1(\omega_r)$  is the first coordinate of  $\omega_r$ .<sup>20</sup>

Let  $\sigma^m = (\sigma_0^m, \sigma_1^m)$  be a symmetric strategy profile. Define the implied ex-ante value function

$$V_{r,\sigma}^{m}(s,t) := \mathbb{E}_{\varepsilon} \Big\{ \sum_{\tau=t}^{\infty} \delta^{\tau-t} \Big[ \pi_{\tau}^{m}(s_{f\tau}, s_{-f,\tau}) - c_{\tau}^{m}(a_{f\tau}, s_{f\tau}) + \varepsilon_{f\tau}(a_{\tau}) - \varphi r \mathbf{1} \left( T_{m} < \tau, s_{f\tau} < 3 \right) \Big] \Big| r, s, t; \sigma \Big\}$$

where  $\mathbb{E}_{\varepsilon}$  indicates that the expectation is taken over the sequence of  $\varepsilon$ 's for all firms; firms' states evolve according to  $(\sigma_0^m, \sigma_1^m)$ .

Symmetry implies restrictions on  $\sigma_0, \sigma_1, V_0, V_1$ . For example, for a regulated firm, it is irrelevant whether  $s_- = (3, 2, 1)$  or  $s_- = (1, 2, 3)$ . Therefore  $V_1(s_1, 3, 2, 1) = V_1(s_1, 1, 2, 3)$  and similarly for the policy function  $\sigma_1$ . Similar restrictions apply to unregulated firms. Furthermore, symmetry implies that  $V_0$  and  $V_1$  are equal for some

<sup>&</sup>lt;sup>20</sup>The idiosyncratic nature of the regulated firm's technology is the reason why I don't define the state variable to be given by the number of competitors with each technology. The model could be equivalently represented in that way, but given that it is necessary to keep track of the regulated firm's technology, it is simpler to keep track of all firms technologies and impose the appropriate symmetry conditions.

states. For example, suppose that  $s_j = s_r \ge 3$ . Then  $V_0(s_j, s_r, s_-) = V_1(s_r, s_j, s_-)$ . Symmetry implies further restrictions on value and policy functions. Appendix C presents all of those restrictions and how they're used to efficiently represent firms' state spaces.

Finally, note that the recursive characterization of Markov Perfect Equilibria (e.g., Doraszelski and Escobar (2010)) implies that  $\{\sigma_0^m, \sigma_1^m\}$  is a MPE if and only if

$$\sigma_r^m(s,t,\varepsilon) = \operatorname*{argmax}_{a \in A(s_f)} \left\{ \pi_t^m(s_{ft}, s_{-f,t}) - c_t^m(a, s_f) + \delta \mathbb{E}_{\varepsilon_{-f}} \left[ V_{r,\sigma}^m\left(a, s_{-f}', t+1\right) | r, s, t \right] + \varepsilon(a) \right\}$$
(2)

where, for firms  $h \neq f$ ,  $s'_h = \sigma^m_{r_h}(s, t, \varepsilon_h)$  and the expectation is with respect to the shocks  $\varepsilon_h$  of firms  $h \neq f$ .

#### 3.2 Quasi-Stationary Markov Perfect Equilibria

As discussed above, there are two sources of non-stationarity in this environment. First, flow profits and cost parameters change over time. Second, coverage requirements imply that firms' policy functions respond to the proximity of the requirement expiration date  $T_m$ . In this subsection, I introduce assumptions that accommodate these two sources of non-stationarity, but impose a degree of stationarity.

The specification of entry and technology upgrade costs assumes that those costs eventually stabilize.<sup>21</sup> I will assume the same of flow profits. Specifically, I assume that flow profits vary in a way known to the firms from the start of my sample until the beginning of 2018, after which they stabilize. I then make two assumptions regarding equilibrium behavior. First, after parameters have stabilized and the expiration date of the coverage requirement has passed, behavior doesn't depend on the date anymore. Second, the same is true if parameters have stabilized and the committed firm has satisfied its commitment.

Formally, I focus on *Quasi-stationary Symmetric Markov Perfect Equilibria*, defined below. Let  $T_{\theta}$  denote the earliest time period such that flow profits and costs do not vary after  $T_{\theta}$ .

**Definition 1.** A Symmetric Markov Perfect Equilibrium  $(\sigma_0, \sigma_1)$  is said to be *quasi-stationary* if there exist functions  $\tilde{\sigma}_r(s, \varepsilon), r \in \{0, 1\}$ , such that, if either

- (i)  $t \ge \max\{T_m + 1, T_\theta\}$ , or
- (ii)  $t \geq T_{\theta}$  and  $s_r \geq 3$ ,

<sup>&</sup>lt;sup>21</sup>Entry and technology upgrade costs vary between the early and the late phases, but do not change after that.

then  $\sigma_r(s, t, \varepsilon) = \tilde{\sigma}_r(s, \varepsilon)$ .

I assume throughout that the data is generated by a Quasi-Stationary Symmetric Markov Perfect Equilibrium. Note that this imposes restrictions on value functions over time. For example, if  $t \ge T_{\theta}$  and  $s_i = s_r \ge 3$ , then  $V_1(s_r, s_i, s_{-}, t) =$  $V_0(s_i, s_r, s_-, t+1)$ . Essentially, the model has a non-stationary phase followed by a stationary phase. Models of technology adoption must somehow contend with the fact that the demand for and costs of adopting a new technology vary over time. One way of dealing with the time-varying nature of demand and costs that appears in the literature is to assume a finite horizon and solve the game played by firms via backward induction; see, e.g., Igami (2017). That method raises the issue of assigning continuation values to different industry states in the final time period. In Igami (2017), that is done by assuming that the state of the industry doesn't change after the terminal period, and computing the implied discounted stream of profits. Quasi-stationarity instead assumes that firms will keep playing the entry and technology upgrade game forever, so that firms' continuation values are given by the equilibrium value function in the relevant states. The empirical feasibility of this assumption rests on observing a cross-section of markets.

#### 3.3 Modeling Flow Profits

It is not uncommon in applications of dynamic games for flow profits to be derived from an estimated demand system paired with an assumption on firms' pricing behavior. Following that route would require data on available plans, their prices, and consumers' choices from the available plans. Unfortunately, such data is not available in my setting. I thus follow a different approach. Suppose that consumer *i* in market *m* with demographic characteristics  $x_i$  chooses what carrier to subscribe to, what technology to use, and how much to spend on mobile telecommunications services,  $e_i$ . Let  $\sigma_{fgt}(s, H)$  be the resulting market share of firm-technology pair (f, g) in period *t* when the industry state is *s* and the distribution of demographics is *H*; a model for  $\sigma_{fgt}$  will be specified below. Let *M* be the size of the market and, as before, let  $s_f$  be firm *f*'s state.<sup>22</sup> Finally, denote by  $\mathbb{E}[e_i|g]$  the expectation of consumers' expenditures  $e_i$ , conditional on a consumer choosing technology  $g.^{23}$  Firms' profits are then given

<sup>&</sup>lt;sup>22</sup>I set the market size to be twice the population of the municipality. The number of mobile telecommunications subscriptions in Brazil is larger than the population.

<sup>&</sup>lt;sup>23</sup>Here I condition only on the chosen technology, and not on the firm identity, because firms are assumed throughout to be symmetric.

by<sup>24</sup>:

$$\pi_t(s_f, s_{-f}, H) = M \sum_{g \in s_f} \sigma_{fgt}(s, H) \left(\mathbb{E}_t[e_i|g] + \psi\right)$$

$$= M \sum_{g \in s_f} \sigma_{fgt}(s, H) \left(\int \mathbb{E}[e_i|g, x_i] dH_t(x_i|g) + \psi\right)$$
(3)

The summation over  $g \in s_f$  indicates that we sum over all technologies offered by firm  $f: \{g : 0 < g \leq s_f\}$ . The parameter  $\psi$  captures revenues that the expenditure model may fail to account for and marginal costs of serving customers. In estimation, I will allow  $\psi$  to vary by groups of markets; see section 4 for details. Note that in equation (3), the conditional distribution  $H_t(x_i|g)$  is indexed by t. That is because consumers' preferences over technologies are allowed to vary over time (as indicated by the t subscripts in  $\sigma_{fgt}$ ), so that the distribution of demographics conditional on technology choice also varies over time.

The main data limitation I face is that I never observe consumer expenditures together with their technology (and carrier) choices. I will therefore make the following assumption:

Assumption 1.  $\mathbb{E}[e_i|g, x_i] = \mathbb{E}[e_i|x_i].$ 

This assumption says that conditional on individual characteristics  $x_i$ , consumer expenditure is mean independent of the technology chosen by that consumer. This is, admittedly, a strong assumption. It would hold, e.g., in a world in which consumers pay per usage (a popular model in Brazil), and technology doesn't affect usage. This assumption would fail if better technologies induce consumers to utilize more data. Assumption 1 would thus be untenable if we were dealing with a population that uses high-bandwith applications. Because we are dealing with small, rural municipalities in Brazil, the assumption is more palatable. Importantly, note that Assumption 1 does not imply that consumers that subscribe to different technologies will spend (on average) the same amount, for individuals with different demographic characteristics are still allowed to sort into different technologies.

<sup>&</sup>lt;sup>24</sup>The expression in the right hand side of 3 is an approximation. Firms' profits are equal to  $\sum_{g \in s_f} \sum_{i \in fg} e_i$ , where the summations are over the technologies offered by firm f and over individuals i subscribing to firm-technology pair (f, g). This approximation holds in the sense that the difference between firms' profits and the right hand side of equation 3 is  $O_p(\sqrt{M})$ , whereas the included term is O(M). This implies that the approximation error becomes negligible relative to the included term for large M. This approximation is analogous to the (implicit) approximation to profit functions used routinely in supply and demand models in empirical industrial organization.

Assumption 1 and equation 3 imply that

$$\pi_t(s,H) = M \sum_{g \in s_f} \sigma_{fgt}(s,H) \left( \int \mathbb{E}[e_i|x_i] dH_t(x_i|g) + \psi \right)$$
(4)

I model  $\sigma_{fgt}(s, H)$  as arising from a nested logit model. Specifically, consumer *i*'s utility of subscribing to firm-technology pair j = (f, g) in market m and year  $\tau$  is given by<sup>25</sup>

$$u_{ijm\tau} = \underbrace{\gamma_{r(m),p(\tau)} + \mu_{g(j),p(\tau)} + \beta_{g(j),p(\tau)} y_{m\tau} + \theta_{g(j),p(\tau)} d_{m\tau}}_{v_{g(j)m\tau}} + \xi_{jm\tau} + \zeta_{im\tau}(\lambda) + (1-\lambda)\varepsilon_{ijm\tau}$$
(5)

where r(m) is the state of municiality m,  $p(\tau)$  is the phase (early or late) associated with year  $\tau$ ,  $y_{m\tau}$  is GDP per capita, and  $d_{m\tau}$  is population density.<sup>26</sup> The term  $\xi_{jm\tau}$  is an unobserved product characteristic,  $\zeta_{im\tau}(\sigma)$  is a disturbance common to all goods other than the outside good, and  $\varepsilon_{ijm\tau}$  is a Type 1 Extreme Value shock. The parameter  $\lambda$  is the nesting parameter, and  $\zeta_{im\tau}(\lambda)$  has the unique distribution such that  $[\zeta_{im\tau}(\lambda) + (1 - \lambda)\varepsilon_{ijm\tau}]$  also has an extreme value distribution (see Cardell (1997)).

In equation (5),  $\gamma_{r(m),p(\tau)}$  is a state-phase fixed effect meant to capture variation in the share of the outside good;  $\mu_{g(j),p(\tau)}$  is a technology-phase fixed effect, which captures changes in the popularity of each technology over time; and the effect of income and population density on consumer preferences is also allowed to vary by technology and phase.

The distributional assumptions above imply that market shares are given by

$$\sigma_{jm\tau}(s, v_{m\tau}, \xi_{m\tau}) = \frac{e^{(v_{g(j)m\tau} + \xi_{jm\tau})/(1-\lambda)}}{D} \times \frac{D^{1-\lambda}}{1+D^{1-\lambda}}$$
(6)

where  $v_{m,\tau}$  is a vector collecting the  $v_{gm\tau}$ ,  $\xi_{m\tau}$  is a vector similarly defined, and  $D := \sum_{j \in s} e^{(v_{g(j)m\tau} + \xi_{jm\tau})/(1-\lambda)}$ , where the summation is over the products offered in the market, which are encoded in the industry state s. The predicted quantity of subscribers is  $M\sigma_{jm\tau}(s)$ .

<sup>&</sup>lt;sup>25</sup>I specify equation 5 at the year level because the demographic variables in it are observed with that frequency. A period in the model, which corresponds to six months, is mapped to its corresponding year and the choice model introduced in the text is used to compute market shares.

<sup>&</sup>lt;sup>26</sup>Ideally,  $y_i$  should be used in equation 5. That would add one more integration in the estimation routine. Doing so is work in progress. In the analysis that follows, when calculating H(x|g), I will treat the coefficient on  $y_{m\tau}$  as the effect of an individual's income on her utility.

It remains to model  $\mathbb{E}[e_i|x_i]$ . I assume that individual *i*'s,  $e_i$ , is given by

$$\log(e_i) = \alpha_{r(i)u} + \alpha_1 \log(y_i) + \alpha_2 n_i + \eta_i \tag{7}$$

In equation (7), r(i) indicates *i*'s state of residence; *u* indicates whether the municipality is classified as urban or rural by the Census;  $y_i$  is income;  $n_i$  is the number of residents in *i*'s household; and  $\eta_i$  is an error term that is uncorrelated with the included regressors. We now have all the ingredients needed to compute firms' profits in equation 4, except for the distribution  $H(x_i|g)$ . I obtain that distribution using the technology choice model outlined above and Census data on municipality-level demographics; for details, see section 4.

The final aspect of the model is an assumption regarding the distribution of  $\xi_{jm\tau}$ . I introduce this assumption to deal with the fact that the I observe the quantities of subscribers at different levels of geographic granularity over time; see section 4 for details.

**Assumption 2.** Let c(m) denote the area-code that municipality m belongs to. The unobserved product characteristic  $\xi_{jm\tau}$  satisfies

$$\xi_{jm\tau} = \xi_{jc(m)\tau} + \eta_{jm\tau}$$

where  $\eta_{jm\tau} \stackrel{iid}{\sim} F$ .

Assumption 2 says that  $\xi_{jm\tau}$  can be decomposed into a random variable that varies only with area-code, on which I place no restrictions, and another RV that varies across municipalities within an area-code, that I assume is *iid* with some unrestricted distribution *F*.

Under Assumption 2, an argument relying on a large number of municipalities within an area-code implies that

$$\sigma_{jc\tau} = \sum_{m \in c} \omega_m \int \sigma_{jm\tau}(s_{m\tau}, v_{m\tau}, \xi_{c(m),\tau}, \eta_{m\tau}; \theta) dF(\eta_{m\tau})$$
(8)

holds approximately.<sup>27</sup> In equation (8),  $\omega_m$  is the fraction of the population in area-code c in municipality m. I will use equation (8) in estimation; see section 4.

<sup>&</sup>lt;sup>27</sup>See Appendix **??** for details.

### **4** Identification and Estimation

I start this section by discussing the estimation of the flow profit function in subsection 4.1. In subsection 4.2 I discuss the estimation of the dynamic parameters of the model, i.e., the entry and upgrade costs and the fine for non-compliance with the regulation.

#### 4.1 Estimation of the Flow Profit Function

The flow profit function is given by equation (4). Computing profits requires four objects:  $\sigma_{fgt}(s, H)$ ,  $\mathbb{E}[e_i|x_i]$ ,  $H_t(x_i|g)$ , and  $\psi$ . In this subsection, I discuss the estimation of the first three of these objects.

The first task is to estimate the parameters underlying the market share terms,  $\sigma_{fgt}(s, H)$ . Here I have to deal with the fact the data on mobile subscriptions come at different levels of geographic granularity over time. First, equation (6) implies the usual analytical nested logit inversion (see Berry (1994)):

$$\log(s_{jmt}) - \log(s_{0mt}) = v_{g(j)mt} + \lambda \log(s_{j|\mathcal{J}_{mt}}) + \xi_{jmt}$$
(9)

where  $\log(s_{j|\mathcal{J}_{mt}})$  is the share of good j in the total number of subscriptions in the market. This equation yields  $\xi_{jmt}$  as a function of data and parameters,  $\xi_{jmt}(\theta)$ . I interact  $\xi_{jmt}(\theta)$  with instruments to form moment conditions  $\mathbb{E}[\xi_{jmt}(\theta)Z_{imt}^1] = 0$ .

The intuition for the identification of the nesting parameter  $\lambda$  is similar to that in Berry and Waldfogel (1999). The nesting parameter determines the extent of business stealing when a new product enters the market. If we can exogenously vary the number of products in the market, we learn the value of  $\lambda$  by observing the effect on the aggregate share of the goods in the market. Following this intuition, I use as instruments for  $\log(s_{j|\mathcal{J}_{mt}})$ , the logarithm of the area of municipality m, and dummies for whether or not the municipality is one of the regulated ones, interacted with the regulation deadline. The area of a municipality increases the cost of providing service, and thus reduces the number of products in the market. Regulated municipalities with early regulation deadlines. The identifying assumption is that the regulated municipalities with later deadlines. The identifying assumption is that the regulation deadlines are uncorrelated with unobservable product characteristics in 2019. I also use the demographic variables in  $v_{jmt}$  as instruments.

The moments discussed above are informative about the nesting parameter and preference parameters in the later period of the data, but not about preference parameters in the earlier period of the data. To construct additional moments to identify those parameters, I leverage assumption 2 and equation (8). Equation (8), repeated here for

convenience, states that market shares at the area-code level are approximately given by

$$\sigma_{jct} = \sum_{m \in c} \omega_m \int \sigma_{jmt}(s_{mt}, v_{mt}, \xi_{c(m),t}, \eta_{mt}; \theta) dF(\eta_{mt})$$
(10)

Equating observed market shares at the area-code level with their predicted counterparts, given by the right hand side of equation 11, allows one to solve for  $\xi_{jct}$  as a function of all the utility parameters. These structural error terms,  $\xi_{jct}(\theta)$ , could then be interacted with instruments to form moment conditions of the form  $\mathbb{E}[\xi_{jct}(\theta)Z_{jct}^2] = 0$ . The one hindrance to that approach is the integration with respect to  $F(\eta_{jmt})$ . Here, again, assumption 2 offers a solution. Given any vector of structural parameters,  $\theta$ , equation (9) gives us  $\xi_{jmt}(\theta)$ . We can then make use of assumption 2 to recover  $\eta_{jmt}(\theta)$ , which gives us an empirical distribution of  $\eta_{jmt}$  given  $\theta$ ,  $F(\eta; \theta)$ . In this way, the integration in equation (11) can be performed for any guess of  $\theta$  by sampling from the implied  $F(\eta; \theta)$ , and moment conditions can be formed as outlined above.

To summarise the preceding discussion, the steps involved in evaluating the GMM objective function for a given value of  $\theta$  are as follows. First, use equation (9) to obtain  $\xi_{jmt}(\theta)$ . Second, use assumption 2 to obtain  $\eta_{jmt}(\theta)$ . Third, solve for  $\xi_{jct}(\theta)$  from

$$s_{jct} = \sum_{m \in c} \omega_m \frac{1}{N_s} \sum_{i=1}^{N_s} \sigma_{jmt}(s_{mt}, v_{mt}, \xi_{c(m),t}, \eta_i; \theta)$$
(11)

where  $s_{jct}$  is the observed market share of firm-technology pair j in area-code c and period t,  $\eta_i$  is a vector of  $|\mathcal{J}_{mt}|$  independent draws from  $F(\eta; \theta)$  and  $N_s$  is the number of simulation draws. Fourth, interact  $\xi_{jmt}$  with  $Z_{jmt}^1$  and  $\xi_{jct}$  with  $Z_{jct}^2$  and average, to get sample analogs of the moment conditions discussed above; call these sample analogs  $\bar{g}^1(\theta)$  and  $\bar{g}^2(\theta)$ , respectively. For a chosen weight matrix W, the GMM objective is then given by

$$J(\theta) := \left(\bar{g}^1(\theta)' \ \bar{g}^2(\theta)'\right) W \begin{pmatrix} \bar{g}^1(\theta) \\ \bar{g}^2(\theta) \end{pmatrix}$$
(12)

The GMM estimator is, as usual,  $\hat{\theta} := \operatorname{argmin}_{\theta} J(\theta)$ . I have discussed the instruments  $Z_{jct}^1$  above. The instruments  $Z_{jct}^2$  used in estimation are the population-weighted averages of the demographics included in  $v_{gmt}$ . I use the identity matrix as the weighting matrix in estimation.

The term  $\mathbb{E}[e_i|x_i]$  in equation (4) is calculated from equation (7), which is estimated by ordinaty least squares using the Household Budget Survey. From (7) it follows that  $\mathbb{E}[e_{im}|x_i] = \exp(\alpha_{r(m)u} + \alpha_2 n_i)y_i^{\alpha}\mathbb{E}[\exp(\eta_{im})|x_i]$ . I assume that  $\exp(\eta_{im})$  is mean independent of  $x_i$  and estimate  $\mathbb{E}[\exp(\eta_{im})]$  using the residuals from equation (7). The last ingredient needed to use equation (4) is the conditional distribution  $H(x_i|g)$ . By Bayes' rule,

$$h(x_i|g) = \frac{\sigma(g|x_i)h(x_i)}{\int \sigma(g|x_i')h(x_i')dx_i'}$$
(13)

The term  $\sigma(g|x_i)$  is derived from the technology choice model; the unconditional distribution of  $x_i$  comes from the Census data. I obtain  $h(x_i|g)$  by drawing a uniform random sample from the municipality-level Census data, computing  $\sigma(g|x_i)$  for each drawn  $x_i$ , and calculating  $\sigma(g|x_i) / \sum_i \sigma(g|x_j)$ .

The final object in equation (4) is the parameter  $\psi$ . I will allow the value of  $\psi$  to vary across five groups of municipalities. Those groups are determined in the following heuristic way. First, I project the number of firm-technology pairs in municipality m and period t onto municipality and time dummies. Next, I run a linear regression of the estimated municipality fixed effects on averages over time of the municipality characteristics included in the structural model. The residuals from these regressions can be thought of as time-invariant unobserved factors that determine the number of products in a market, and hence are related to profitability in that market. I group municipalities according to the quintiles of the distribution of these residuals and estimate a  $\psi$  for each group. These parameters will be estimated together with the dynamic parameters of the model. That is the topic of the next subsection.<sup>28</sup>

#### 4.2 Identification and Estimation of Dynamic Parameters

The flow payoffs of the dynamic game introduced in the previous section are linear in the structural parameters. For this class of models (dynamic games with linear flow payoffs), it is possible to show that structural parameters are identified if conditional choice probabilities are identified.<sup>29</sup> The requirement that conditional choice probabilities be identified excludes from this result general models with unobserved state variables However, this result encompasses models where the unobserved state variables possess a group structure and that group structure can be recovered from the

<sup>&</sup>lt;sup>28</sup>The heuristic procedure discussed in this subsection is related to approaches taken by Collard-Wexler (2013) and Sanches, Silva-Junior, and Srisuma (2018) to account for unobserved heterogeneity. A recent literature in econometrics has introduced methods to deal with group fixed effects in panel data and structural models. On this, see, e.g., Bonhomme and Manresa (2015), Bonhomme, Lamadon, and Manresa (2017), and Cheng, Schorfheide, and Shao (2019). It is possible that those methods can be adapted to deal with group-level unobserved heterogeneity in dynamic games. A more common approach of dealing with market-level unobserved heterogeneity would be to apply an EM-type algorithm. That approach, however, would require making the strong assumption that unobserved heterogeneity is independent across markets, which seems unlikely in the present case.

<sup>&</sup>lt;sup>29</sup>This is a known result, see, e.g., Aguirregabiria and Nevo (2013). I review the argument here for completeness.

data in a first stage, as is assumed here.

The conditional value functions inherit the linearity from the flow payoffs: there exist functions  $f_{rt,P^m}(a, s)$  and  $g_{rt,P^m}(a, s, z)$  such that

$$\frac{v_{r,t}^{m}(a,s)}{\sigma} = f_{rt,P^{m}}(a,s) + g_{rt,P^{m}}(a,s,z)\sigma^{-1}\Psi$$

where  $\Psi$  is a vector collecting all structural parameters (see Appendix D for details). This fact can be used to establish identification.

Since the idiosyncratic errors follow a Type 1 Extreme Value distribution, the conditional choice probabilities have the logit form:

$$P^{m}(a|s, r, t) = \frac{\exp(v_{r,t}^{m}(a, s)/\sigma)}{\sum_{a' \in A(s_{f})} \exp(v_{r,t}^{m}(a', s)/\sigma)}$$

We can apply the usual logit inversion to this equation to obtain:

$$\ln(P^{m}(a|s,r,t)) - \ln(P^{m}(s_{f}|s,r,t)) = \frac{v_{r,t}^{m}(a,s)}{\sigma} - \frac{v_{r,t}^{m}(s_{f},s)}{\sigma}$$

Using the linear representation of the conditional value functions we can then write

$$\ln(P^{m}(a|s,r,t)) - \ln(P^{m}(s_{f}|s,r,t)) - f_{rt,P^{m}}(a,s) - f_{rt,P^{m}}(s_{f},s) = \left[g_{rt,P^{m}}(a,s,z) - g_{rt,P^{m}}(s_{f},s,z)\right]' \frac{\psi}{\sigma}$$
(14)

Equation (14) leads to an OLS-like formula for  $\psi/\sigma$ .<sup>30</sup>

The intuition for identification is that the structural parameters are identified by exogenous variation in ( $\pi_m$ ,  $z_m$ , s, r, t) and the fact that we observe how firms respond to this variation (i.e., we observe conditional choice probabilities). One can, for example, entertain the thought experiment of varying one of the exogenous covariates and observing how the behavior of firms changes. If we vary the distribution of income, for example, flow profits will vary; the extent to which firms respond in their entry and upgrade behavior is informative about the costs of such actions.<sup>31</sup> The fine parameters

<sup>&</sup>lt;sup>30</sup>This argument has used market-specific CCPs  $P^m$ . This is not necessarily inconsistent with the typical assumption that a unique equilibrium is played in the data, as one can simply enlarge the state space to include market-level characteristics and define policy functions on that domain. Either way, those CCPs must be estimable from data, and we therefore require that the equilibria played in the data (or the unique equilibrium defined on an enlarged state space) vary continusouly with the market-level characteristics.

<sup>&</sup>lt;sup>31</sup>Although useful, this intuition is slightly imprecise. When we vary the distribution of income, the endogenous conditional choice probabilities  $P^m$  will also change, thus changing the other terms in  $w_{rt,P^m}^m(a,s,z_m)$ . This makes clear that functional form assumptions play a role in obtaining identification in dynamic games, which is why all empirical models in this literature are tightly parameterized.

ter  $\varphi$  is identified by the difference in behavior between regulated and unregulated firms. Time variation provides additional variation to identify  $\varphi$ . Intuitively, for small  $\varphi$  the behavior of regulated firms will change only slightly as the regulation deadline approaches; large  $\varphi$ , on the other hand, should lead to larger changes in behavior.

#### 4.3 Estimation

I apply the Nested Pseudo Likelihood (NPL) algorithm of Aguirregabiria and Mira (2007) to estimate the dynamic parameters. In light of the results of Pesendorfer and Schmidt-Dengler (2010), my choice of estimator requires some justification. A popular alternative is to use a two-step estimator, e.g. Bajari, Benkard, and Levin (2007), Pakes et al. (2007) or Pesendorfer and Schmidt-Dengler (2008). These estimators all proceed by flexibly estimating policy functions in a first stage and then using those policy functions to construct a second-stage objective function that is then minimized to yield structural estimates. Because my model features substantial non-stationarity, it would be challenging to obtain flexible and accurate first stage estimates of policy functions. For this reason, I opt to use an estimator that makes full use of the already imposed structural assumptions.

As is well known, the computational cost of the maximum likelihood estimator is prohibitive in the case of dynamic games. I thus adopt Aguirregabiria and Mira (2007). An alternative that was recently proposed is Dearing and Blevins (2019). The estimator proposed by Dearing and Blevins (2019) enjoys good theoretical properties. In particular, it is guaranteed to converge, thus overcoming the main issue raised of NPL raised by Pesendorfer and Schmidt-Dengler (2010). However, the algorithm in Dearing and Blevins (2019) requires solving large systems of linear equations, which renders its application to the empirical setting in this paper substantially more costly than Aguirregabiria and Mira (2007).

A Nested Pseudo Likelihood (NPL) fixed point is a pair  $(\hat{\theta}, \{\tilde{P}^m\}_m)$  that satisfies

(i) 
$$\theta = \operatorname{argmax}_{\theta} \sum_{m,t,f} \ln \Psi(a_{fmt} | s_{mt}, r_{fm}, t, m; \theta, P^m)$$

(ii) 
$$\tilde{P}^m = \Psi(\tilde{P}^m; \tilde{\theta})$$
 for all  $m$ 

The NPL *estimator* is the NPL fixed point with the maximum value of the pseudolikelihood. The set of NPL fixed points is known to be non-empty. However, it need not be a singleton. This implies that the researcher must explore the parameter space to ensure that the pseudo-likelihood is being maximized in the set of NPL fixed points.

In practice, one finds NPL fixed points via an iterative algorithm. Starting with a guess for CCPs,  $\{\tilde{P}^m\}_m$ , the implied pseudo likelihood is maximized (see condition (i)

above). One then uses the resulting guess for  $\theta$  to update firms' CCPs (see condition (ii)) above. These two steps are repeated until the CCPs or the structural parameters converge.

### 5 Estimation Results

Table 5 presents the estimates of the static parameters, those in the market shares and expenditure functions. The results show that the market share of 4G is, in both the early and the later periods, the most responsive to income, suggesting that richer individuals have higher demand for high-bandwith uses of mobile communications. The market share of 4G also grows the most with population density in the earlier part of the sample. This is consistent with individuals in more densely populated areas having more social connections and therefore having higher demand for faster connectivity. Surprisingly, this pattern is more muted in the later part of the sample. Estimates of the expenditure model show that richer individuals spend more on mobile telecommunications, as one would expect. Mobile expenses also increase in the number of residents in the household; this is consistent with the notion that individuals in larger families have more reason to communicate and are therefore more active users of mobile telecommunications services.

Table 6 displays estimates of the dynamic parameters: entry costs, technology upgrade costs, the cost of non-compliance with the regulation, the standard deviation of the cost shocks, and the unobservable profitability parameters,  $\psi$ . The costs associated with the introduction of 3G are found to be essentially constant over time. In contrast, the costs of introducing 4G decrease sharply, driven by the coefficient on the municipality's area.<sup>32</sup> Lastly, the fine is found to be very substantial: 6.89 million BRL, which is just over 40% of the median entry cost.

### 6 Counterfactual Analysis

The counterfactual exercises I conduct in this section directly address the questions posed in the beginning of the paper. In subsection 6.1, I use the model to analyze the effect of coverage requirements on the time to introduction of 3G technology. I also use the model to quantify the cost that the regulation imposes on firms and to decompose the total effect of coverage requirements into a direct effect on the regulated firm and indirect equilibrium effects. In subsection 6.2, I use the model to evaluate alternative

<sup>&</sup>lt;sup>32</sup>In fact, the cost of upgrading to 4G in the later period is found to be negative.

	Market Shares, Early			Marke	et Shares	Expenditures	
Intercept	2G 7.718	3G 6.532	4G	2G 3.740	3G 4.001	4G	
Log Income	0.211	0.421	0.819	-0.205	-0.057	0.366	0.356 (0.003)
Pop Dens.	0.269	0.341	0.423	0.160	0.185	0.213	()
Residents							0.031 (0.002)
$\lambda$		0.628			0.628		
$N R^2$							71 <i>,</i> 994 0.199
State-Phase FEs State-Rural FEs	Yes No	Yes No	Yes No	Yes No	Yes No	Yes No	No Yes

Table 5: Static Parameter Estimates

The first six columns show estimates of the parameters in equation (5), separately by the two phases, early and late. Estimation is based on moment conditions formed using area-code level data for the 2013-2018 period (4,113 observations), and municipality-level data for 2019 (36,290 observations). See section 4 for details on estimation. The last column shows OLS estimates of equation 7. These estimates are based on survey data on consumers' expenses on mobile telecommunications services and demographic characteristics.

Parameter	Estimate	Parameter	Estimate
$\sigma$	2.461	$\varphi$	6.896
$ heta_{e,0}$	19.418	$ heta_{e,Area}$	-0.432
$ heta^{E}_{3G,0}$	7.363	$ heta^{E}_{3G,Area}$	0.721
$\theta^L_{3G,0}$	7.634	$ heta_{3G,Area}^L$	0.750
$ heta^{BE,0}_{4G,0}$	-13.896	$ heta^{E}_{4G,Area}$	3.555
$ heta_{4G,0}^L$	-11.569	$\theta^L_{4G,Area}$	0.865
$\psi_1$	-0.199	$\psi_2$	-0.051
$\psi_3$	-0.013	$\psi_4$	0.014
$\psi_5$	0.067		

Table 6: Dynamic Parameter Estimates

 $\sigma$  is the standard deviation of the cost shocks.  $\varphi$  is the cost of failing to comply with the regulation.  $\theta_{e,0}$  is the entry cost intercept.  $\theta_{e,Area}$  is the coefficient on the logarithm of area in the entry cost function. The remaining parameters are associated with installing 3G and 4G technology. The subscripts 3G and 4G indicate the technology. The subscripts 0, *Area* indicate the intercept and the area term, respectively. The superscripts *E*, *L* indicate the two periods, early and late. For example,  $\theta_{4G,0}^L$  is the intercept of the cost of introducing 4G technology in the later period. The parameters  $\psi_1, \ldots, \psi_5$  are the unobservable profit terms; see equation 3 and the discussion therein.

regulations. Specifically, I consider policies that subsidize the first firm to introduce 3G technology, as well as an intervention that uses coverage requirements as insurance, in the sense that the regulated firm only incurs noncompliance costs in case no firm introduces 3G technology.

#### 6.1 The Effect of Coverage Requirements

To quantify the effect of coverage requirements on the time to introduction of 3G technology and firms' ex-ante expected profits, I use the estimated model to simulate data under two alternative regulatory regimes. First, I solve the game and simulate data under the estimated fine  $\hat{\varphi}$ . Second, I solve the game and simulate data setting  $\varphi = 0$ , i.e., with no regulation. I simulate 250 paths of play for each municipality under each of these two regulatory regimes.

First, I compute the fraction of the 250 simulations in which some firm introduced 3G technology by December 2019. Figure 4 shows the distribution of those probabilities across municipalities. The figure shows that 66.45% of the municipalities in the sample would have had access to 3G technology with at least 75% probability. For 88.75% of the municipalities, the probability of having 3G access by December 2019 is at least 50%. This suggests that for most municipalities, market forces would most likely than not be sufficient to guarantee provision of 3G service. Figure 4 also shows that for 11.25% of municipalities, the probability of having 3G service by December 2019 is less than 50%. In these municipalities, market forces are insufficient to guarantee service provision. All the municipalities that have no service in the beginning of the data are in this group.

The results above may suggest that the regulation has limited effect, given that the probability of having 3G service by December 2019 is high for most municipalities. However, the regulation turns out to have non-negligible effects on the time to introduction of 3G technology. I use the models with and without regulation to simulate data until 2023. For each municipality and regulatory regime, I calculate the average number of years before the introduction of 3G technology or better.<sup>33</sup> Figure 5 shows the resulting distributions. In the figure, the label "Status quo" refers to setting the fine to  $\hat{\varphi}$ . The label "No regulation" corresponds to setting the fine to 0. As can be seen from the figure, the regulation reduces the average time to 3G introduction significantly – by 1.15 years, on average. The regulation also considerably reduces the dispersion in

<sup>&</sup>lt;sup>33</sup>In those instances in which 3G is not introduced by the end of the simulated data, I set the time to 3G introduction equal to the length of the simulated sample. This implies that the numbers I present on the effect of the regulation are, in some cases, a lower bound.

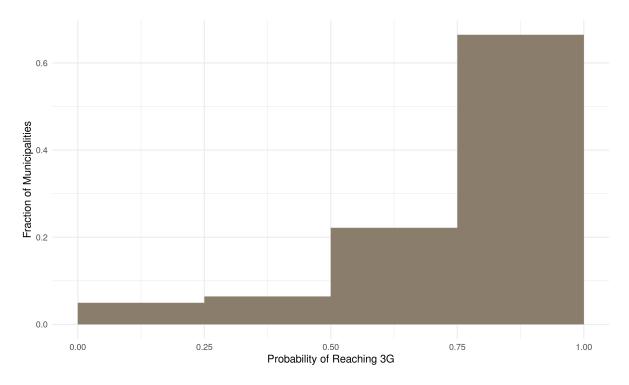


Figure 4: Distribution of the Probability of Reaching 3G by December 2019 Without Regulation.

the time to introduction of 3G, mostly by eliminating a long right tail present in the absence of regulation.

Figure 6 shows the same information in a different way. For each municipality, I compute the accelleration in the introduction of 3G due to the regulation. Figure 6 plots the resulting distribution across municipalities. The effects are concentrated between 0 and 2 years, though there is a long right tail, consisting of the most vulnerable markets. For 4 municipalities, the regulation delays the introduction of 3G, though those effects are quantitatively small. In those cases, the equilibrium effects dominate the direct effect of the regulation.

To further understand the determinants of the effects of the regulation, I investigate how the time to 3G introduction in the absence of regulation and the acceleration afforded by coverage requirements relate to observable market characteristics. Specifically, I project the time to introduction of 3G with no regulation and the acceleration induced by coverage requirements onto observable market characteristics and variables that capture the initial market structure. I restrict attention to the municipalities that did not have 3G in the beginning of the sample. Table 7 reports the results.

The dependent variable in column 1 of table 7 is the time to 3G introduction without regulation, measured in years, and the explanatory variables are a municipality's

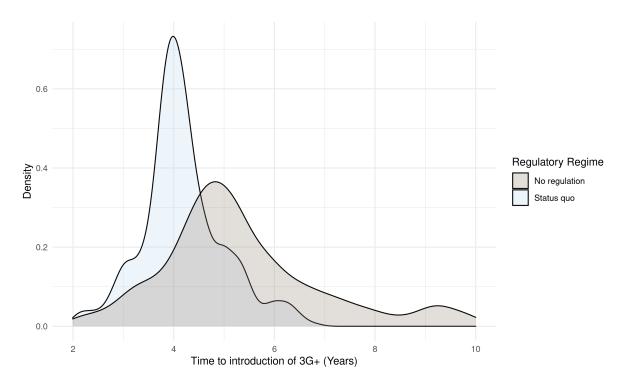


Figure 5: Distribution of the time to introduction of 3G technology or better under alternative regulatory regimes.

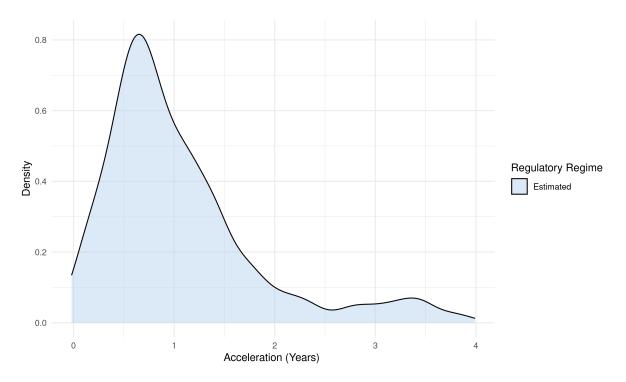


Figure 6: How much faster is the introduction of 3G+ under regulation?

		Depe	ndent variable:	
	Time to 3G	Speedup	No. Entrants	<b>Regulation</b> Cost
	(1)	(2)	(3)	(4)
Log GDP	-0.324***	-0.221***	0.031***	
	(0.052)	(0.027)	(0.007)	
Log Population	-0.363***	-0.130***	0.084***	
	(0.070)	(0.037)	(0.010)	
Log Area	0.415***	0.253***	-0.049***	
	(0.025)	(0.013)	(0.004)	
No. Firms $t = 0$	-1.630***	-0.319***		
	(0.082)	(0.051)		
Regulated Active $t = 0$		0.293***	$-0.884^{***}$	
0		(0.032)	(0.009)	
Regulated				2.889***
				(0.050)
Active				0.127***
				(0.017)
Regulated * Active				-2.732***
0				(0.041)
Constant				0.007***
				(0.001)
Group FEs	Yes	Yes	Yes	No
Observations	689	665	665	3,008
$\mathbb{R}^2$	0.797	0.747	0.948	0.863
Adjusted R <sup>2</sup>	0.795	0.744	0.947	0.863
Note:			*p<0.1; **	p<0.05; ***p<0.01

# Table 7: Explaining Time to Adoption and the Effect of Regulation

GDP per capita, population, and area, as well as the number of firms in the beginning of the data.<sup>34</sup> The results show that the time to 3G introduction without regulation is decreasing in a municipality's GDP per capita and in its population, and it is increasing in a municipality's area. Moreover, the time to 3G introduction is decreasing in the number of firms in the market in t = 0. These results are all intuitive: firms are more likely to enter and upgrade their technologies in richer, more populous, and smaller markets; since incumbents have a lower cost of introducing 3G than potential entrants, a larger initial number of firms leads to faster 3G introduction.

The second column in table 7 models the acceleration in the introduction of 3G generated by coverage requirements, measured in years, as a function of the same variables included in column 1, and additionally a dummy for whether the regulated firm was active in the market in the beginning of the data.<sup>35</sup> The coefficients on the market characteristics and the number of firms show the same pattern as column 1, i.e., markets where, in the absence of regulation, 3G would be introduced faster also experience a smaller acceleration, as one might expect. Lastly, the estimates imply that regulating an incumbent leads to a larger acceleration than regulating a potential entrant, of just under four months.

The third column in table 7 reports a regression of the average number of entrants in a market, computed from simulations at the end of 2022, on market characteristics and whether the regulated firm was active in t = 0. Regulating an incumbent instead of a potential entrant reduces the average number of entrants by just under 0.9. The sign is expected, as a regulated potential entrant has to enter the market. The coefficient is less than one in absolute value because regulating an incumbent implies that there is one more unregulated potential entrant, and thus more entry that is not due to the regulation. Moreover, unregulated potential entrants may be more likely to enter the market when an incumbent firm is regulated, because they expect less future competition. I will show below, however, that for most markets the magnitude of this mechanism is not of first order importance.

Next, I use the model to calculate the cost that the regulation imposes on firms.<sup>36</sup> Solving the dynamic game under the estimated fine and under no regulation, I obtain, for each municipality, firms' ex-ante expected profits under those two regimes. The

<sup>&</sup>lt;sup>34</sup>I take averages over time of these municipality characteristics. The sample is restricted to those markets that do not have 3G service in the beginning of the data.

<sup>&</sup>lt;sup>35</sup>To aid in the interpretability of the coefficient on the dummy, this regression and the one in the third column further restrict attention to those municipalities that had at least one active firm in the beginning of the data

<sup>&</sup>lt;sup>36</sup>Note that in the real world part of this cost is borne by the government, via reduced revenue in spectrum auctions.

cost of the regulation is the aggregate difference in firms' ex-ante expected in profits in the no-regulation and the status-quo regimes:

Regulation Cost = 
$$\sum_{m} \sum_{f} \left( V_{\varphi=0}^{m}(s_{f0}, s_{-f0}, t=0) - V_{\varphi=\hat{\varphi}}^{m}(r_{f}, s_{f0}, s_{-f0}, t=0) \right)$$

where  $V_{\varphi}^{m}(\omega)$  is the firm's ex-ante expected profit in municipality m and state  $\omega$  when the fine is set to  $\varphi$ .<sup>37</sup> For the set of municipalities used in estimation, I calculate that the cost of the regulation amounts to 2.11 billion 2010 BRL, or 1.2 billion 2010 USD.<sup>38</sup> This amounts to 24.14% of firms' aggregate ex-ante expected profits with no regulation.

To understand the sources of these costs, the last column in table 7 reports estimates of a regression of the municipality-firm-specific regulation cost,  $V^m_{arphi=0}(s_{f0},s_{-f0},t)$  $0) - V^m_{\varphi=\hat{\varphi}}(r_f, s_{f0}, s_{-f0}, t=0)$ , onto a dummy for whether or not the firm is regulated, a dummy for whether or not the firm was active in the market in t = 0, and their interaction. The estimates show that for unregulated potential entrants (i.e., Regulated = 0and Active = 0), the cost of the regulation is essentially zero. It is slightly positive because the regulation leads to a more competitive market when these potential entrants do enter, thus reducing their profits. As discussed in more detail below, that effect is small, which explains the small cost imposed on these firms. The cost for unregulated active firms is larger, because these firms are directly affected by the extra competition brought about by coverage requirements. On average, these firms lose about 134,000 USD because of the regulation, which is equivalent to 3.63% of their ex-ante expected profit without regulation. That effect depends on the technology of the incumbent firm: unregulated firms with 2G technology lose about 96,000 USD (3.22% of their profits), whereas unregulated firms with 3G technology lose about 346,000 USD (4.51% of their profits).

The bulk of the regulation costs falls on the regulated firms. The cost imposed on regulated firms with 2G technology is, on average, 291,000 USD, or 9.74% of their profits under no regulation. The cost imposed on regulated firms that are not active in the market is very substantial: it is equal to 2.90 million USD, which is almost 13 times their ex-ante expected profits of (on average) 223,000 USD. This cost comes from the fact that these firms are forced to enter the market when they might have chosen not to do so. Overall, 84.62% of the costs imposed on firms come from those imposed on regulated potential entrants; 9.71% come from regulated incumbents; and the re-

<sup>&</sup>lt;sup>37</sup>Note that in the first term,  $V_{\varphi=0}^m(s_{f0}, s_{-f0}, t = 0)$ , I do not include the regulation indicator  $r_f$  as an argument because there is no regulation in that case;  $r_f$  does appear as an argument in the second term.

<sup>&</sup>lt;sup>38</sup>This conversion uses the average exchange rate in 2010.

maining 5.67% come from unregulated firms, i.e., they amount to costs stemming from competition effects of the regulation.

The fact that the regulation costs imposed on incumbents is substantially smaller than that imposed on potential entrants, combined with the fact that regulating incumbents leads to a faster introduction of 3G (see column 2 of table 7), may suggest that coverage requirements should be imposed on active firms. In practice, the extent to which this policy can be pursued, however, is limited by concerns of competitive neutrality. Such a policy would also provide poor incentives to firms, as entering new markets would make a firm more likely to be regulated in the future. Furthermore, this policy has an opportunity cost that is illustrated by column 3 of table 7: imposing the coverage requirement on an incumbent leads to less competition in the market than imposing the requirement on a potential entrant.<sup>39</sup> For the imposition of coverage requirements on potential entrants to be better, in aggregate welfare terms, than imposing those requirements on incumbents, the added competition (which is of 0.88 firms on average, according to table 7) has to generate an increase in consumer surplus of 8.60 BRL per month. This is equal to 50.65% of the average predicted expenditure for this set of municipalities.<sup>40</sup>

I close this subsection on the effects of coverage requirements by decomposing these effects into direct effects on the regulated firm and indirect equilibrium effects. To do so, I proceed in three steps. First, I solve the game and simulate data in the absence of regulation. I then solve for the regulated firm's optimal policy given the estimated fine and holding the policy functions of the unregulated firms fixed at their equilibrium policies without regulation. Next, I solve for the Markov Perfect Equilibrium under regulation. The difference between the time to adoption under the equilibrium policies with regulation and the time to adoption when only the regulated firm responds to the regulation gives the desired equilibrium effects.

Figure 7 shows the distribution, across municipalities, of the equilibrium effects. Most of the values are positive: the equilibrium adjustment leads to a longer time to introduction of 3G, relative to the case when only the regulated firm adjusts its behavior to the policy. This reflects the reduced incentives to enter and upgrade faced

<sup>&</sup>lt;sup>39</sup>Column 1 of table 7 suggests a second cost. Imposing the coverage requirement on a potential entrant may also accelerate the introduction of subsequent technologies. Below I investigate the effects of alternative coverage requirements on the adoption of 4G (which hasn't been directly regulated for this set of municipalities).

<sup>&</sup>lt;sup>40</sup>This number is obtained by dividing the added cost from imposing the requirement on a potential entrant (assuming a single active firm in the market and setting the costs on inactive firms to zero) by the average population in the subsample of municipalities that don't have 3G in the beginning of the sample, which is 4,689, using the discount factor used in the model to arrive at a monthly gain in consumer surplus.

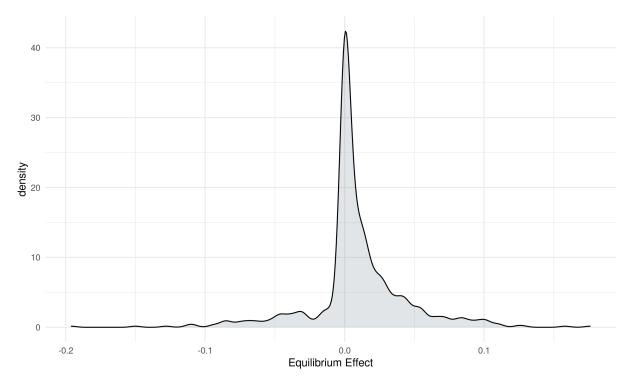


Figure 7: Equilibrium Effects

by unregulated firms, resulting from the increased future competition generated by the regulation. Quantitatively, however, the equilibrium effects are very small. The total effects of the policy are therefore almost entirely explained by the direct effects on the regulated firm.

### 6.2 Alternative Regulatory Interventions

The final question posed in the beginning of this paper was whether we can design more effective regulation than coverage requirements. As before, I am mostly concerned with two dimensions of a policy's effect: to what extent it accelerates the introduction of the new technologies, and the cost of adoption of these new technologies. I will also highlight the effect of different policies on market structure. In this subsection, I evaluate two alternative forms of intervention: using coverage requirements as "insurance" against lack of service, and subsidizing the first firm to introduce 3G.

#### 6.2.1 Coverage Requirements as Insurance

The regulation currently in place consists of tasking one firm with introducing 3G technology by a given date. If that firm fails to do so, it incurs a cost of non-compliance.

An alternative implementation of coverage requirements would be to impose costs on the regulated firm only if no firm provides 3G by the regulation deadline. This implementation would achieve introduction of 3G by the regulation deadline (assuming sufficiently strong enforcement), and it would also have benefits relative to the current implementation of the regulation. First, it would reduce the cost imposed on the regulated firm, because if some other firm chooses to introduce 3G, the regulated firm would not be subject to the requirement anymore. Second, this implementation of coverage requirements would do away with negative entry deterrence effects. However, given the results above showing that the equilibrium effects of the regulation are quantitatively small, this benefit should also be small.

Results and discussion to be added.

#### 6.2.2 Subsidizing the Introduction of 3G

The large estimated cost of non-compliance and the counterfactual results above show that coverage requirements provide strong incentives for the regulated firm to introduce 3G. These strong incentives ensure service provision. However, they come at the cost of forcing a firm to enter a market or upgrade its technology when it might not have done so in the absence of regulation. The analysis above established that these costs are substantial, especially when the regulated firm is not active in the market.

A policy that treats firms symmetrically, instead of focusing on a single firm, may save on these costs. The intuition is simple. By providing the same incentive to all firms, the firm that will eventually choose to introduce the new technology will tend to be the most cost-efficient one.

Motivated by this reasoning, in this section I evalute a regulation that subsidizes the first firm to introduce 3G technology or better. I denote the subsidy by  $\beta$ . If more than one firm introduces the new technology, those firms split the subsidy equally. Therefore, I add the following term to firms' flow profits for each state of the game

$$\beta \times \mathbf{1}\underbrace{\left\{\left(\max_{f'} s_{f'}\right) < 3 \le a_f\right\}}_{\text{Subsidy is paid}} \times \underbrace{\sum_{n=0}^{3} \mathbb{P}\left(\left(\sum_{f' \ne f} \mathbf{1}\{a_{f'} \ge 3\}\right) = n\right) \times \frac{1}{1+n}}_{\text{Expected fraction of the subsidy}}$$

where the probabilities in this expression are derived from the ensuing equilibrium behavior.

I experiment with two subsidy designs. I start with a budget given by

Budget = 
$$\sum_{m} \sum_{f} \left( V_{\varphi=0}^{m}(s_{f0}, s_{-f0}, t=0) - V_{\varphi=\hat{\varphi}}^{m}(r_{f}, s_{f0}, s_{-f0}, t=0) \right)$$
 (15)

This amount is simply the aggregate cost of the regulation. Note that firms would be willing to pay this amount to move from the status quo world to a world with a subsidy. In that sense, the subsidies considered below are self-financed.

I start by simply splitting the budget in equation (15) equally across municipalities. Figure 8 shows the resulting acceleration in the introduction of 3G technology obtained under coverage requirements (labeled "status quo" in the figure) and the subsidy. The average effect is very similar; the subsidy accelerates the introduction of 3G by 1.07 years on average, relative to 1.15 years under coverage requirements. The subsidy generates larger accelerations for 63.71% of the municipalities. As figure 8 shows, relative to coverage requirements, the subsidy eliminates some small effects, but also loses some large ones. The large effects lost by the subsidy come precisely from those municipalities that would experience relatively late introduction of 3G in the absence of regulation. Consider, for example, those municipalities where coverage requirements generate an acceleration in the introduction of 3G of one year and a half or more. The average time to introduction of 3G without regulation in these municipalities is almost three years more than in the remaining municipalities. This is a set of municipalities where the introduction of 3G is relatively unprofitable, and the homogeneous subsidy provides less incentives for 3G introduction in these municipalities than coverage requirements do. For this set of municipalities, the subsidy leads to 3G introduction 1.2 years later than coverage requirements, on average.

The municipalities where coverage requirements generate small accelerations (less than 6 months) are relatively competitive municipalities. The average number of firms in t = 0 in those municipalities is 1.54, relative to 0.96 in the remaining municipalities. The introduction of 3G in these municipalities in the absence of regulation is relatively fast: just under 3.5 years, on average, compared to just over under 5.5 years in the other municipalities. In these markets, the effect of the subsidy is very close to the mean effect, so that these markets are moved from the left tail of the "Status Quo" distribution if figure 8 to the middle of the subsidy distribution. In summary, relative to coverage requirements, a flat subsidy increases the acceleration of 3G introduction in some localities where there seems to be little need for regulation, and has smaller effects in some municipalities where regulation seems to be particularly important.

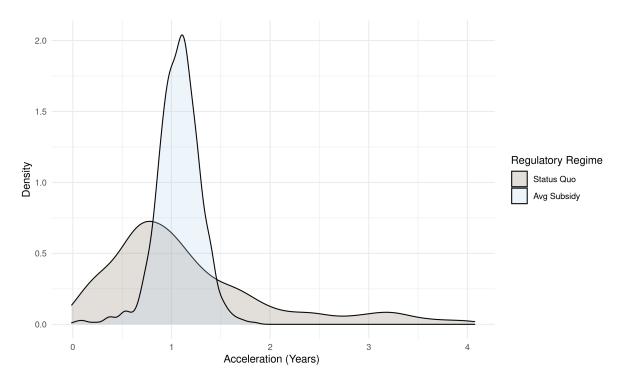


Figure 8: Acceleration of 3G Introduction Under Coverage Requirements and Subsidy

This point is shown clearly in figure 9. The figure shows a scatterplot of the time to introduction of 3G technology in the absence of regulation against the effects of coverage requirements and the flat subsidy. Each dot in the figure is one municipality. For the case of coverage requirements, we see a positive correlation: the regulation has stronger effects in those markets where, in the absence of intervention, it would take longest for 3G to be introduced. The flat subsidy does not display the same correlation. In fact, its effects are smallest for the most vulnerable municipalities.

In light of these results, I consider a subsidization policy that allocates a larger share of the budget towards the most vulnerable municipalities. Specifically, let  $\tau_m$ be the time for 3G introduction in municipality m in the absence of regulation and let f be a positive and increasing real function. Allocate to municipality m the fraction  $f(\tau_m)/\sum_{m'} f(\tau'_m)$  of the budget specified in equation (15). The more convex fis, the stronger the targeting towards the most vulnerable municipalities. For the results below, I set  $f(\tau) = \tau^{3/2}$ .<sup>41</sup> Figure 10 shows the results. This subsidy leads to a

<sup>&</sup>lt;sup>41</sup>This subsidy design relies on  $\tau_m$ , and one may thus be concerned that its informational requirements are substantial. However, note that the results in table 7 show that a substantial portion of the variation in  $\tau_m$  is explained by observables. Therefore, it might be possible to design a subsidy with similar properties that relies only on data that is available to regulators.

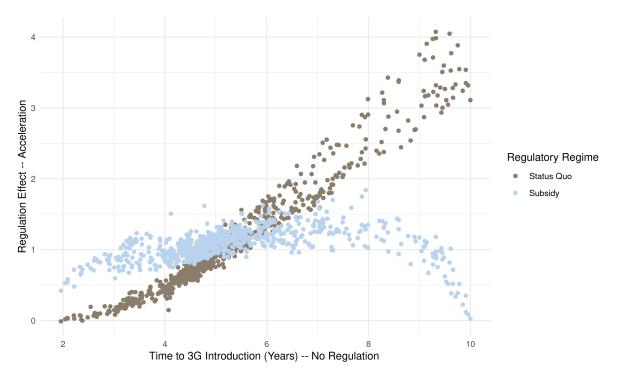


Figure 9: Targeting Properties of Coverage Requirements and a Flat Subsidy

larger acceleration in the roll-out of 3G: 1.27 years relative to 1.15 years under coverage requirements. As shown in the figure, the municipality-specific subsidy restores the desired positive correlation between the effect of the regulation and the time to 3G introduction in the absence of regulation. In fact, this subsidy leads to larger accelerations in the roll-out of 3G in the most vulnerable municipalities than do coverage requirements. This comes at the expense of slightly smaller effects in those municipalities that even in the absence of regulation obtain access to 3G technology relatively quickly. The optimal way to navigate this trade-off (e.g., the optimal choice of exponent in  $f(\tau)$ ) depends on the relative changes in consumer surplus in those two groups of municipalities, which can't be quantified with the limited data available in this study.

Finally, firms substantially benefit from the municipality-specific subsidy relative to coverage requirements.<sup>42</sup> Firms' ex-ante aggregate expected profits grow by 659 million USD, after accounting for their financing of the subsidy (as per equation (15)); this amounts to 17.46% of firms' aggregate profits under coverage requirements. These gains essentially come from reallocating the introduction of the new technology from

<sup>&</sup>lt;sup>42</sup>Similar results hold for the flat subsidy.

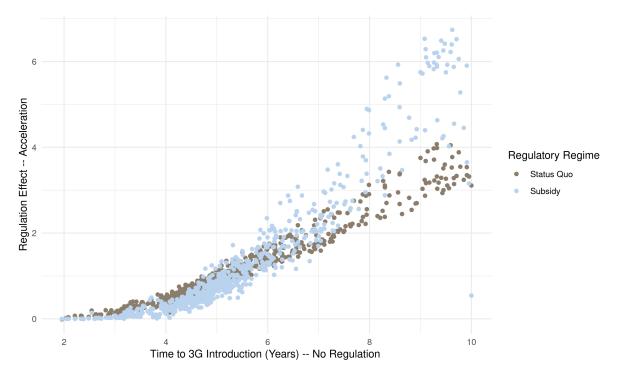


Figure 10: Targeting Properties of Coverage Requirements and a Municipality-Specific Subsidy

inactive and regulated firms, who have to pay entry costs to enter the market, to incumbents, who only pay technology installation costs.

This reallocation leads to a more cost-efficient technology roll-out, but at the expense of reduced competition in the market. The subsidy leads to entry of 0.51 firms, on average, by the end of 2022. In contrast, coverage requirements lead to entry of 0.84 firms. Moreover, this difference is entirely driven by those markets where the regulated firm is a potential entrant, which are the source of the cost savings discussed above. The average number of entrants in these markets, under coverage requirements, is 1.32, whereas it is equal to 0.57 under the subsidy. In the remaining markets, those where the regulated firm is an incumbent, coverage requirements result in entry of 0.42 firms, on average; the subsidy results in entry of 0.46 firms. In those markets where a potential entrant is subject to the coverage requirement, the coverage requirement would be more desirable than the subsidy if the added competition from one additional firm generated additional consumer surplus of 7.35 BRL; that amounts to 44.69% of consumers' average expenditures in those markets.

### 7 Conclusion

Concerns regarding lack of service provision are widespread and so is regulatory intervention. This paper studies the effect of coverage requirements, a common form of regulation in the mobile telecommunications industry, on the speed of roll-out of new technologies, market structure, and firms' profits. To do so, I use new mobile technology availability data from Brazil to estimate a dynamic model of entry and technology upgrade under regulation.

Counterfactual simulations show that in the absence of regulation, third generation mobile telecommunications technology would have been introduced just over one year later, on average. This faster introduction comes at a high cost: firms' ex-ante expected profits are 24% lower under the existing regulation than they would have been in its absence. I also use the model to evaluate alternative policies. In particular, I find that a policy that subsidizes the first firm to introduce 3G technology, by an amount that the firms themselves would be willing to finance, achieves a slightly larger acceleration of the introduction of 3G and leads to more cost-efficient patterns of roll-out, likely increasing aggregate welfare. These findings have immediate implications for the design of regulation in mobile telecommunications markets, and potentially to other markets where universal service is also a concern.

Some interesting and related questions are not addressed in this paper. First, though my results are informative for the design of regulation, data limitations preclude me from conducting a complete welfare analysis. It would be interesting to combine data such as the one used in this paper with detailed price and quantity data to compare the gains in consumer surplus from having earlier access to new technologies and the regulatory costs imposed on firms. Second, my analysis abstracted away from spatial correlation in firms' costs. It would also be interesting, though challending, to study the introduction of new mobile telecommunications technologies while modeling geographic cost interdependencies. These interesting and challenging topics are left for future research.

# References

- Aguirregabiria, V., & Mira, P. (2007). Sequential estimation of dynamic discrete games. *Econometrica*, 75(1), 1–53.
- Aguirregabiria, V., & Nevo, A. (2013). Recent developments in empirical io: Dynamic demand and dynamic games. *Advances in Economics and Econometrics*, *3*, 53–122.
- Aker, J. C., & Mbiti, I. M. (2010). Mobile phones and economic development in Africa. *Journal of economic Perspectives*, 24(3), 207–32.
- Akerman, A., Gaarder, I., & Mogstad, M. (2015). The skill complementarity of broadband internet. *The Quarterly Journal of Economics*, 130(4), 1781–1824.
- Armstrong, M. (2001). Access pricing, bypass, and universal service. *American Economic Review*, 91(2), 297–301.
- Bajari, P., Benkard, C. L., & Levin, J. (2007). Estimating dynamic models of imperfect competition. *Econometrica*, *75*(5), 1331–1370.
- Berry, S. T. (1994). Estimating discrete-choice models of product differentiation. *The RAND Journal of Economics*, 242–262.
- Berry, S. T., & Waldfogel, J. (1999). Free entry and social inefficiency in radio broadcasting. *The RAND Journal of Economics*, 30(3), 397–420.
- Bertschek, I., & Niebel, T. (2016). Mobile and more productive? firm-level evidence on the productivity effects of mobile internet use. *Telecommunications Policy*, 40(9), 888–898.
- Björkegren, D. (2019). The adoption of network goods: Evidence from the spread of mobile phones in rwanda. *The Review of Economic Studies*, *86*(3), 1033–1060.
- Bonhomme, S., Lamadon, T., & Manresa, E. (2017). Discretizing unobserved heterogeneity. University of Chicago, Becker Friedman Institute for Economics Working Paper(2019-16).
- Bonhomme, S., & Manresa, E. (2015). Grouped patterns of heterogeneity in panel data. *Econometrica*, *83*(3), 1147–1184.
- Cardell, N. S. (1997). Variance components structures for the extreme-value and logistic distributions with application to models of heterogeneity. *Econometric Theory*, 185–213.
- Cheng, X., Schorfheide, F., & Shao, P. (2019). *Clustering for multi-dimensional heterogeneity.* Manuscript.
- Choné, P., Flochel, L., & Perrot, A. (2002). Allocating and funding universal service obligations in a competitive market. *International Journal of Industrial Organiza-tion*, 20(9), 1247–1276.

- Collard-Wexler, A. (2013). Demand fluctuations in the ready-mix concrete industry. *Econometrica*, *81*(3), 1003–1037.
- Czernich, N., Falck, O., Kretschmer, T., & Woessmann, L. (2011). Broadband infrastructure and economic growth. *The Economic Journal*, 121(552), 505–532.
- Dearing, A., & Blevins, J. R. (2019). Efficient and convergent sequential pseudo-likelihood estimation of dynamic discrete games. *arXiv preprint arXiv:*1912.10488.
- Doraszelski, U., & Escobar, J. F. (2010). A theory of regular markov perfect equilibria in dynamic stochastic games: Genericity, stability, and purification. *Theoretical Economics*, *5*(3), 369–402.
- Doraszelski, U., & Judd, K. L. (2012). Avoiding the curse of dimensionality in dynamic stochastic games. *Quantitative Economics*, *3*(1), 53–93.
- Dunne, T., Klimek, S. D., Roberts, M. J., & Xu, D. Y. (2013). Entry, exit, and the determinants of market structure. *The RAND Journal of Economics*, 44(3), 462–487.
- Ericson, R., & Pakes, A. (1995). Markov-perfect industry dynamics: A framework for empirical work. *The Review of economic studies*, 62(1), 53–82.
- Gowrisankaran, G., Lucarelli, C., Schmidt-Dengler, P., & Town, R. (2011). Government policy and the dynamics of market structure: Evidence from critical access hospitals. *manuscript. University of Arizona*.
- Igami, M. (2017). Estimating the innovator's dilemma: Structural analysis of creative destruction in the hard disk drive industry, 1981–1998. *Journal of Political Economy*, 125(3), 798–847.
- Jack, W., & Suri, T. (2014). Risk sharing and transactions costs: Evidence from kenya's mobile money revolution. *American Economic Review*, 104(1), 183–223.
- Jensen, R. (2007). The digital provide: Information (technology), market performance, and welfare in the South Indian fisheries sector. *The Quarterly Journal of Economics*, 122(3), 879–924.
- Pakes, A., Ostrovsky, M., & Berry, S. (2007). Simple estimators for the parameters of discrete dynamic games (with entry/exit examples). *the RAND Journal of Economics*, 38(2), 373–399.
- Pesendorfer, M., & Schmidt-Dengler, P. (2008). Asymptotic least squares estimators for dynamic games. *The Review of Economic Studies*, 75(3), 901–928.
- Pesendorfer, M., & Schmidt-Dengler, P. (2010). Sequential estimation of dynamic discrete games: A comment. *Econometrica*, 78(2), 833–842.
- Roller, L.-H., & Waverman, L. (2001). Telecommunications infrastructure and economic development: A simultaneous approach. *American economic review*, 91(4), 909–923.

- Ryan, S. P. (2012). The costs of environmental regulation in a concentrated industry. *Econometrica*, *80*(3), 1019–1061.
- Sanches, F., Silva-Junior, D., & Srisuma, S. (2018). Banking privatization and market structure in brazil: a dynamic structural analysis. *The RAND Journal of Economics*, *49*(4), 936–963.
- Schmidt-Dengler, P. (2006). The timing of new technology adoption: The case of mri. *Manuscript, London School of Economics*.
- Valletti, T. M., Hoernig, S., & Barros, P. P. (2002). Universal service and entry: The role of uniform pricing and coverage constraints. *Journal of Regulatory Economics*, 21(2), 169–190.
- Wu, T. (2010). The master switch: The rise and fall of information empires. Vintage.

# Appendix

# Appendix A Regulation and Delay in the Fudenberg-Tirole Model

#### A.1 The Model

There are two firms. Firm 1 is an incumbent and firm 2 a potential entrant. Time is continuous and the discount rate is r. Firm 1 initially operates as a monopolist with constant marginal cost  $\bar{c}$ . At any point in time  $t \ge 0$ , firms can adopt a technology with constant marginal cost  $\underline{c}$ . Adopting this technology at time t costs C(t), where C(t) > 0, C'(t) < 0 and C''(t) > 0, for all  $t \ge 0$ .

Let  $p^m(c)$  and  $\pi^m(c)$  be, respectively, the monopoly price and profit when marginal cost is c. I focus on the case in which the innovation is *non-drastic*, i.e.,  $p^m(\underline{c}) \geq \overline{c}$ . If both firms are in the market, they compete à la Bertrand. Let  $\pi^d(c, c')$  be a firm's profit when its cost is c and its competitor's cost is c'. Under the assumption of a non-drastic innovation and Bertrand competition,  $\pi^d$  satisfies

$$\pi^d(\underline{c}, \overline{c}) = (\overline{c} - \underline{c})D(\overline{c}), \quad \pi^d(\overline{c}, \underline{c}) = 0 \quad \text{and} \quad \pi^d(c, c) = 0 \quad \forall c$$

Firms' strategies specify their decisions to adopt or not the new technology as a function of t and their competitor's technology<sup>43</sup>. Note that due to the Bertrand assumption, a firm will never adopt the new technology after its competitor has adopted, as they would incur the positive adoption cost but their flow profits would stay at zero.

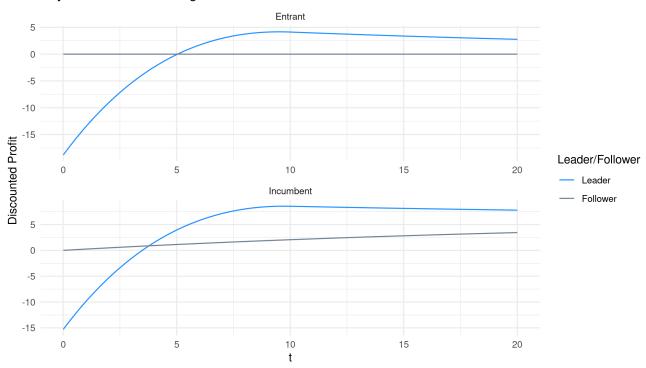
If the incumbent is first to adopt at date  $t_1$ , its overall profit is

$$L_1(t_1) = \int_0^{t_1} \pi^m(\bar{c}) e^{-rt} dt + \int_{t_1}^\infty \pi^m(\underline{c}) e^{-rt} dt - C(t_1) e^{-rt_1}$$
(16)

If the incumbent is preempted at date  $t_2$ , its present discounted profit is

$$F_1(t_2) = \int_0^{t_2} \pi^m(\bar{c}) e^{-rt} dt$$
(17)

<sup>&</sup>lt;sup>43</sup>The discussion here is somewhat informal. Fudenberg and Tirole 1985 provide a careful description of appropriate strategies for this game. Their analysis is far from trivial.



Payoffs in the Fudenberg-Tirole Game

Figure 11: Payoffs in the Fudenberg-Tirole Model.

If the entrant is first to adopt at date  $t_2$ , its overall profit is

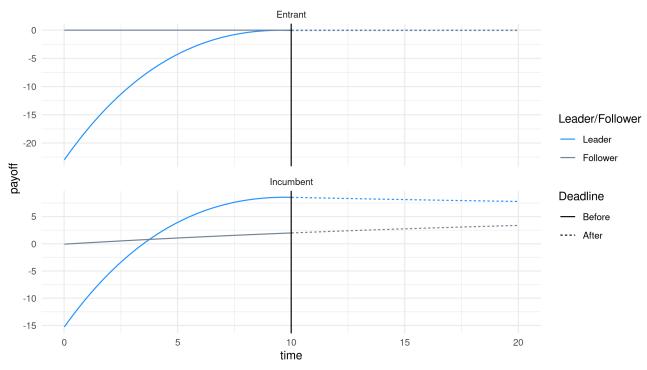
$$L_2(t_2) = \int_{t_2}^{\infty} \pi^d(\underline{c}, \overline{c}) e^{-rt} dt - C(t_2) e^{-rt_2}$$
(18)

Finally, it the entrant is preempted at time  $t_1$ , its profit is given by  $F_2(t_1) = 0$ .

Figure 11 plots the functions<sup>44</sup>  $L_1$ ,  $F_1$ ,  $L_2$ ,  $F_2$ . That figure is sufficient to determine the equilibrium outcome of the game<sup>45</sup>. Let  $t_2^*$  be defined by  $F_2(t_2) = L_2(t_2)$ . In Figure 11,  $t_2^* \approx 5$ . Firm 2 will not adopt before  $t_2^*$ , as it would prefer to be preempted by firm 1. Knowing this, firm 1 will wait to adopt, as  $L_1(t_1)$  is increasing over  $t_1 < t_2^*$ . Now suppose firm 2 is first to adopt at some  $t_2 > t_2^*$ . Since  $L_1(t_2 > F_1(t_2))$ , firm 1 prefers to adopt at  $t_2 - \varepsilon$ . In equilibrium, firm 1 adopts at  $t_1 = t_2^*$ , and firm 2 never adopts.

<sup>&</sup>lt;sup>44</sup>The specification is as follows. D(p) = 2 - p,  $\bar{c} = 1$ ,  $\underline{c} = 3/4$ ,  $C(t) = \mathbf{1}\{t \le 10\} \left(\frac{t^2}{4} - 5 * t + 25\right) + 0.1$ .

<sup>&</sup>lt;sup>45</sup>But not the equilibrium itself, i.e., the strategy profile.



#### Payoffs in the Fudenberg-Tirole Game with Coverage Requirements

Figure 12: Payoffs in the Fudenberg-Tirole Model with Regulation.

#### A.2 Incorporating Regulation

Now suppose that the incumbent is regulated: is must adopt by some exogenously set deadline  $\tau$ , lest it pay an exorbitant fine. The  $L_i$  and  $F_i$  functions are now defined (for  $t_i \leq \tau$ ) as follows:

$$L_{1}(t_{1}) = \int_{0}^{t_{1}} \pi^{m}(\bar{c})e^{-rt}dt + \int_{t_{1}}^{\infty} \pi^{m}(\underline{c})e^{-rt}dt - C(t_{1})e^{-rt_{1}}$$
(19)  

$$F_{1}(t_{2}) = \int_{0}^{t_{2}} \pi^{m}(\bar{c})e^{-rt}dt - C(\tau)e^{-r\tau}$$
  

$$L_{2}(t_{2}) = \int_{t_{2}}^{\tau} \pi^{d}(\underline{c},\bar{c})e^{-rt}dt - C(t_{2})e^{-rt_{2}}$$
  

$$F_{2}(t_{1}) = 0$$

Figure 12 plots these payoffs for the same parametrization underlying Figure 11, and  $\tau = 10$ . As can be seen from the figure, the fact that the incumbent will adopt the technology at time  $\tau$ , at the latest, eliminates all incentive for the entrant to adopt the new technology. With no need to preempt the entrant, the incumbent is free to delay its own adoption to its most preferred time, which in this example is  $t_1^* \approx 9.7$ . Therefore,

the regulation delays the adoption of the new technology from  $t \approx 5$  to  $t \approx 9.7$ . Of course, if  $\tau < 5$ , the regulation speeds up the adoption of the new technology.

# Appendix B Descriptive Models – Alternative Specifications

This appendix reports alternative specifications of the descriptive models in table 4. In particular, table 8 reports models without group fixed effects, and table 9 reports models that include characteristics of firms' networks in neighboring states. Specifically, it includes dummies for whether or not the firm provides 2G, 3G, and 4G service in any neighboring municipality. Comparing table 8 and table 4 shows the importance of the group fixed effects. Without them in table 8, the competition coefficients are mostly small in absolute value and sometimes positive. That is in stark contrast with the results in table 4, where the competition coefficients are almost all negative and much larger in absolute value. This suggests that the group fixed effects capture important unobserved factors related to how desirable it is to provide service in a given market.

Now let me turn to table 9. The first thing to note is that service in neighboring municipalities is important. The estimated coefficients on 3G service and 4G service are sizeable and precisely estimated. Interestingly, the coefficients on 2G service in neighboring municipalities are negative. This is surprising because these coefficients are relative to not having service in the neighboring municipality. The next thing to observe is the effect of the network variables on the competition coefficients. These effects are mostly small, except perhaps for the number of competitors with 4G technology. Albeit small, the effects are always in the direction of increasing (in absolute value) the estimated competition coefficients. This may suggest that there are unobservable factors that are geographically correlated<sup>46</sup>. Finally, and most importantly for the analysis in this paper, note that the effect of the network variables on the regulation variables is very minor, if it exists at all. This suggests that the regulation variables (in particular, whether or not a firm is regulated) are not correlated with the surrounding network infrastructure.

Table 10 tests the hypothesis of no correlation between a firm's status as the regulated firm and that firm's infrastructure in neighboring markets. The unit of analysis for the models in table 10 is a firm-market pair, and only data from the June 2016 (the first period in the data) is used. The table reports estimation results of a logit model and a linear probability model (included for the sake of interpretability) where the dependent variable is a dummy that takes the value 1 if the firm is regulated, and 0 otherwise. The explanatory variables are a constant and a set of dummies. The vari-

<sup>&</sup>lt;sup>46</sup>Variables that are currently omitted and could potentially be included are variables related to the terrain.

		Dep	pendent varial	ole:	
			Upgrade		
	Out 13-15	Out 16-18	2G 13-15	2G 16-18	3G
	(1)	(2)	(3)	(4)	(5)
Log GDP PC	0.389*** (0.064)	-0.001 (0.081)	0.241*** (0.047)	-0.194*** (0.051)	0.186*** (0.032)
Log Pop.	0.761*** (0.066)	0.728*** (0.086)	0.851*** (0.051)	0.430*** (0.058)	-0.059* (0.035)
Log Area	$-0.123^{***}$ (0.031)	$-0.078^{**}$ (0.040)	$-0.221^{***}$ (0.025)	-0.233*** (0.027)	0.018 (0.019)
Regulated	1.712*** (0.110)	2.111*** (0.126)	2.312*** (0.076)	0.926*** (0.104)	-0.398*** (0.040)
Regulated Competitor - Out	-0.662*** (0.173)	$-1.167^{***}$ (0.284)	0.320** (0.150)	-0.221 (0.162)	-0.137 (0.132)
Regulated Competitor - 2G	-0.021 (0.115)	-0.157 (0.192)	-0.304** (0.120)	$-1.202^{***}$ (0.314)	-2.345*** (0.235)
No. Competitors 2G	-0.044 (0.069)	-0.374*** (0.097)	-0.035 (0.036)	0.137*** (0.049)	$-0.064^{**}$ (0.027)
No. Competitors 3G	-0.269*** (0.090)	$-1.239^{***}$ (0.104)	0.047 (0.047)	-0.001 (0.053)	0.190*** (0.033)
No. Competitors 4G	0.212 (1.031)	$-0.466^{***}$ (0.107)	-1.343* (0.719)	-0.307*** (0.056)	0.411*** (0.034)
Group FE Observations	No 36,230	No 31,620	No 24,753	No 14,002	No 39,923

Table 8: Entry/Upgrade Models – Without group fixed effects	

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

	Dependent variable:				
	Out 13-15	Out 16-18	upgrade 2G 13-15	2G 16-18	3G
	(1)	(2)	(3)	(4)	(5)
Log GDP PC	1.772*** (0.093)	1.116*** (0.120)	0.693*** (0.066)	0.261*** (0.071)	0.323*** (0.039)
Log Pop.	2.537*** (0.106)	2.151*** (0.151)	1.337*** (0.072)	1.081*** (0.085)	0.137*** (0.047)
Log Area	$-0.512^{***}$ (0.038)	$-0.398^{***}$ (0.051)	$-0.294^{***}$ (0.027)	$-0.402^{***}$ (0.031)	-0.063*** (0.020)
Regulated	1.716*** (0.110)	2.269*** (0.130)	2.191*** (0.077)	0.887*** (0.110)	-0.275*** (0.042)
Regulated Competitor - Out	$-0.720^{***}$ (0.173)	-0.997*** (0.285)	0.131 (0.152)	$-0.364^{**}$ (0.168)	-0.100 (0.136)
Regulated Competitor - 2G	0.099 (0.114)	0.023 (0.195)	$-0.487^{***}$ (0.121)	$-1.114^{***}$ (0.319)	-2.155*** (0.236)
No. Competitors 2G	-1.445*** (0.093)	-1.153*** (0.120)	$-0.448^{***}$ (0.056)	-0.339*** (0.069)	-0.154*** (0.039)
No. Competitors 3G	-2.072*** (0.122)	$-2.265^{***}$ (0.146)	$-0.667^{***}$ (0.082)	$-0.741^{***}$ (0.088)	-0.015 (0.050)
No. Competitors 4G	-1.796* (1.036)	$-1.823^{***}$ (0.158)	-2.310*** (0.734)	$-1.275^{***}$ (0.093)	$-0.184^{***}$ (0.051)
Nb. Service 2G	-0.398** (0.157)	$-1.135^{***}$ (0.171)	-0.174 (0.230)	-0.534 (0.328)	-0.047 (0.204)
Nb. Service 3G	1.040*** (0.097)	1.523*** (0.180)	0.601*** (0.063)	0.654*** (0.108)	0.490*** (0.161)
Nb. Service 4G	0.960*** (0.179)	0.495*** (0.097)	0.575*** (0.133)	1.200*** (0.062)	1.640*** (0.043)
Group FE Observations	Yes 36,230	Yes 31,620	Yes 24,753	Yes 14,002	Yes 39,923

# Table 9: Entry/Upgrade Models – With Neighboring Network Info

Note:

\*p < 0.1; \*\*p < 0.05; \*\*\*p < 0.01

able "2G Service" is equal to 1 if the firm provides 2G service in that market; "3G service" is analogously defined. "2G Service Nb." is equal to 1 if the firm provides 2G service in some neighboring market, and "3G Service Nb." is defined similarly. The results show that, conditional on the technologies offered by a firm in the market, which are included in the structural model, its infrastructure in neighboring municipalities has a small effect on the probability that the firm is regulated. The point estimates are in fact negative. These results suggest that there is no cause for concern that the difference in behavior between regulated and unregulated firms, which identifies the fine parameter  $\varphi$  in the structural model, is driven not by the regulation itself but by omitted differences in firms' neighboring infrastructure. Therefore, despite the importance of neighboring infrastructure shown in table 9, I omit these variables from the structural model, as doing so would likely not bias the inference regarding the effects of regulation and would increase the computational burden by several orders of magnitude.

### Appendix C Symmetry Restrictions

The symmetry assumption implies the following restrictions on value functions (and policy functions):

- $V_1(s_1, s_{-1}, t) = V_1(s_1, P(s_{-1}), t)$ , for any permutation *P*.
- $V_0(s_1, s_r, s_-, t) = V_0(s_1, s_r, P(s_-), t)$  for any permutation *P*.
- If  $s_r \ge 3$  and  $\exists j \notin \{1, r\}$  s.t.  $s_j \ge 3$ , then  $V_0(s_1, s_r, s_j, s_k, t) = V_0(s_1, s_j, s_r, s_k, t)$
- If  $s_1, s_r \ge 3$ , then  $V_0(s_1, s_r, s_-) = V_1(s_1, P(s_r, s_-))$  for any *P*.

Add discussion on state space representation.

# Appendix D Conditional Value Functions are Linear in Parameters

In this section I will simplify notation by letting  $\omega$  denote a generic state of the form  $\omega = (t, r, s_f, s_{-f})$ . Flow payoffs, net of the idiosyncratic shock, are given by

$$\pi(\omega) - \varphi r \mathbf{1}\{s_f < 2, T < t\} - c(a, s_f)$$

	Dependent variable:			
	Regulated			
	Logit	LPM		
	(1)	(2)		
2G Service	1.727***	0.237***		
	(0.058)	(0.008)		
3G Service	0.883***	0.194***		
	(0.058)	(0.010)		
2G Service Nb.	$-0.240^{*}$	-0.018		
	(0.125)	(0.016)		
3G Service Nb.	-0.345***	-0.047***		
	(0.052)	(0.008)		
Constant	-2.104***	0.116***		
	(0.117)	(0.015)		
Observations	13,204	13,204		
$\mathbb{R}^2$		0.139		
Adjusted R <sup>2</sup>		0.139		
Note:	*p<0.1; **p<0.05; ***p<0.01			

Table 10: Testing for Selection on Infrastructure in Neighboring Municipalities

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This expression can be written as a linear function of parameters. To see this, first redefine actions *a* and firm technological states  $s_f$  to be vectors indicating the presence of each technology, ordered from 4G to 2G. For example, if a firm offers 3G and 2G, represent  $s_f$  as  $s_f = (0, 1, 1)$ . The deterministic part of costs can then be written as

$$\left[ (a' - s'_f) \otimes (\mathbf{1}\{p(t) = E\}, \mathbf{1}\{p(t) = L\}) \otimes z' \right] \underbrace{(\theta'_{4,E}, \theta'_{3,E}, \theta'_{2,E} + \theta_e, \theta'_{4,L}, \theta'_{3,L}, \theta'_{2,L} + \theta_e)'}_{\theta}$$

Define

$$g(a, \omega, z) := \left(\pi(\omega), \left[ (a - s_f) \otimes (\mathbf{1}\{p(t) = E\}, \mathbf{1}\{p(t) = L\}) \otimes z \right], r\mathbf{1}\{s_f < 2, T < t\} \right)$$

and

$$\Psi := (1, \theta', \varphi)'$$

Then we have

$$\pi(\omega) - \varphi r \mathbf{1}\{s_f < 2, T < t\} - c(a, s_f) = g(a, \omega, z)\Psi$$

The value function satisfies the Bellman equation

$$V(\omega,\varepsilon_f) = \max_{a \in A(s_f)} g(a,\omega,z)\Psi + \varepsilon_f(a) + \delta \sum_{\omega'} V(\omega')F_P(\omega'|\omega,a)$$

where  $F_P$  denotes the state transitions induced by the equilibrium conditional choice probabilities P and

$$V(\omega') := \int V(\omega, \varepsilon_f) dG(\varepsilon_f)$$

Denote the equilibrium policy by  $\sigma^*(s, \varepsilon_f)$ . Then (using  $\sigma^*$  as shorthand for  $\sigma^*(s, \varepsilon_f)$ )

$$V(\omega,\varepsilon_f) = g(\sigma^*,\omega,z)\Psi + \varepsilon_f(\sigma^*) + \delta \sum_{\omega'} V(\omega')F_P(\omega'|\omega,\sigma^*)$$

Integrating both sides of this equation yields

$$V(\omega) = \left(\int g(\sigma^*, \omega, z) dG(\varepsilon_f)\right) \Psi + \int \varepsilon_f(\sigma^*) dG(\varepsilon_f) + \delta \sum_{\omega'} V(\omega') \int F_P(\omega'|\omega, \sigma^*) dG(\varepsilon_f)$$

Let  $\mathcal{C}(a,\omega)$  be the set of shocks  $\varepsilon_f \in \mathbb{R}^{|A(s_f)|}$  such that  $a = \sigma^*(\omega, \varepsilon_f)$ . Then

$$\int g(\sigma^*, \omega, z) dG(\varepsilon_f) = \sum_{a \in A(s_f)} \int_{\mathcal{C}(a,\omega)} g(\sigma^*, \omega, z) dG(\varepsilon_f)$$
$$= \sum_{a \in A(s_f)} g(a, \omega, z) \int_{\mathcal{C}(a,\omega)} dG(\varepsilon_f)$$
$$= \sum_{a \in A(s_f)} g(a, \omega, z) P(a|\omega)$$

where here  $P(a|\omega)$  are the equilibrium conditional choice probabilities.

Similarly,

$$\int P(\omega'|\omega, \sigma^*) dG(\varepsilon_f) = \underbrace{\sum_{a \in A(s_f)} F_P(\omega'|\omega, a) P(a|\omega)}_{F_P(\omega'|\omega)}$$

The term on the right hand side of this equation is simply the probability that the state moves from  $\omega$  to  $\omega'$ , induced by the equilibrium conditional choice probabilities. I will denote that term by  $F_P(\omega'|\omega)$ .

Finally, observe that

$$\int \varepsilon_f(\sigma^*) dG(\varepsilon_f) = \sum_{a \in A(s_f)} \int_{\mathcal{C}(a,\omega)} \varepsilon_f(a) dG(\varepsilon) = \sum_{a \in A(s_f)} P(a|\omega) \mathbb{E}[\varepsilon_f(a)|a = \sigma(\omega,\varepsilon_f)]$$

It is well known that for the Type I Extreme Value distribution,  $\mathbb{E}[\varepsilon_f(a)|a = \sigma(\omega, \varepsilon)] = \sigma(\gamma - \ln P(a|\omega))$ , where  $\gamma$  is the Euler-Mascheroni constant. Therefore

$$\int \varepsilon_f(\sigma^*) dG(\varepsilon_f) = \sigma \sum_{a \in A(s_f)} P(a|\omega)(\gamma - \ln P(a|\omega))$$

Putting these pieces together, we have

$$V(\omega) = \left(\sum_{a} g(a, \omega, z) P(a|\omega)\right) \Psi + \sigma \sum_{a \in A(s_f)} P(a|\omega)(\gamma - \ln P(a|\omega)) + \delta \sum_{\omega'} V(\omega') F_P(\omega'|\omega)$$

or

$$V(\omega) = \mathbb{E}_P[g(a, \omega, z)]\Psi + \sigma\gamma - \sigma\mathbb{E}_P[\ln P(a|\omega)] + \delta F_P(\omega)V$$

where  $\mathbb{E}_P$  denotes an expectation with respect to *a* using the distribution over *a* defined by *P*,  $F_P(\omega)$  is a row vector with the transition probabilities in state  $\omega$ , and *V* a

vector with the value function in each state  $\omega$ .

We can now stack these equations. Let  $M_P$  denote the transition matrix induced by P,  $M = [F_P(\omega'|\omega)]_{\omega,\omega'}$ . Then<sup>47</sup>

$$V = \mathbb{E}_P[g(a, z)]\Psi + \sigma\gamma - \sigma\mathbb{E}_P[\ln P(a)] + \delta M_P V$$

From this equation we obtain

$$V = (I - \delta M_P)^{-1} \Big\{ \mathbb{E}_P[g(a, z)] \Psi + \sigma \gamma - \sigma \mathbb{E}_P[\ln P(a)] \Big\}$$
$$= \sigma K(P) + (I - \delta M_P)^{-1} \mathbb{E}_P[g(a, z)] \Psi$$

where  $K(P) := (I - \delta M_P)^{-1}(\gamma - \mathbb{E}_P[\ln P(a)])$ 

The conditional value function is, by definition,

$$v(a,\omega) = g(a,\omega,z)\Psi + \delta \sum_{\omega'} V(\omega')F_P(\omega'|\omega,a) = g(a,\omega,z)\Psi + \delta F_P(\omega,a)V$$

where  $F_P(\omega, a)$  is the distribution over  $\omega'$  induced by taking action a in state  $\omega$ . Using the result above for V yields

$$\frac{v(a,\omega)}{\sigma} = \delta F_P(\omega,a)K(P) + \left\{ g(a,\omega,z) + \delta F_P(\omega,a)(I-\delta M_P)^{-1} \mathbb{E}_P[g(a,z)] \right\} \sigma^{-1} \Psi$$

<sup>&</sup>lt;sup>47</sup>In this equation, it is to be understood that the scalar  $\sigma\gamma$  is added to all coordinates. The  $\omega$ -th coordinate of  $\mathbb{E}_P[g(a, z)]$  is equal to  $\sum_{a \in A(s_f)} g(a, \omega, z) P(a|\omega)$ . Similarly for  $\mathbb{E}_P[\ln P(a)]$